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Fuzzy Logic-Based Approach to Behavioral Economics: Mathematical Modeling of Consumer Decision-Making

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Abstract

Even the mathematics that some people want to apply to this decision-making is limited, as consumer decisions are filled with uncertainty, subjective evaluation, and cognitive biases. In this paper, we develop and apply a model entitled Fuzzy Logic-Based Decision Model to evaluate consumers' preferences regarding smartphone selections in the presence of uncertainty. The proposed model utilizes fuzzy set theory, linguistic variables and IF-THEN rule-based inference systems to capture consumer evaluations on price, battery life, and brand reputation. Using four smartphone models as a case study, it shows the model's ability to embrace vagueness and ambiguity in consumers' choices. From the outcomes Smartphone A proved to be the max preferable smartphone followed by Smartphone C and least favorable smartphones were B and D. The results are consistent with the predictions of behavioral economics, which suggest that consumers weigh competing attributes in their choices, rather than optimizing on a single dimension. IoT data was used to apply the fuzzy logic model to consumer behavior using Fuzzy Logic based Classification system The presented model was able to capture these trade-offs accurately and thus proved to be a realistic and flexible approach for the analysis of consumer behaviour. So, the study develops a Fuzzy Logic Heuristics based model that helps to overcome these limitations and provides the constructs which Fuzzy Logic itself overcomes in conventional attitudes towards consumer behaviour. The future extensions involving, market research applications, neuro-fuzzy systems, and machine learning integration and temporal modeling are suggested in this paper for the use of both academic research and in practice by e-commerce platforms and product recommendation system.

Keywords: *Fuzzy Logic, Consumer Decision-Making, Uncertainty Handling, Behavioral Economics, Preference Modeling, Product Selection, Fuzzy Inference System, Linguistic Variables, Bounded Rationality, Machine Learning Integration.*

Introduction

Background and Motivation

Behavioral economics bridges the gap between traditional economic models and real-world

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consumer behavior, incorporating psychological insights into economic decision-making (Kahneman & Tversky, 1979; Mohammad, 2025). Traditional economic theories, such as utility maximization and rational choice, assume that consumers make decisions based on well-defined preferences and complete information. However, in reality, human decision-making is often influenced by cognitive biases, emotions, and heuristics, leading to deviations from purely rational behavior (Thaler, 1980; Galdolage et al., 2024).

Fuzzy logic provides a robust mathematical framework for modeling the inherent uncertainty and vagueness in human decision-making. Fuzzy sets represent a fundamental innovation that enables a more accurate modeling of decision making processes by accounting for the subjectivity and imprecision inherent to consumer preferences (Zadeh 1965; Mohammad et al., 2025c).

Overview of Behavioral Economics

Behavioral economics encompasses psychological truths in economic models in order to make sense of human decision-making (Simon, 1955). Some key concepts include:

- **Bounded Rationality:** Consumers make decisions with limited cognitive resources, leading to satisficing rather than optimizing behavior.
- **Prospect Theory:** Consumers evaluate outcomes relative to a reference point, displaying loss aversion (Kahneman & Tversky, 1979; Mohammad et al., 2025e).
- **Heuristics and Biases:** Decision-making is influenced by mental shortcuts, such as anchoring, availability, and representativeness heuristics (Tversky & Kahneman, 1974; Mohammad et al., 2025a).

Need for Mathematical Modeling in Consumer Decision-Making

Consumer choices involve multiple subjective factors, including preferences, risk perception, and emotional responses. Conventional mathematical models are based on crisp sets and probability theory, which may not fully capture the vagueness in human behavior (Zadeh, 1975). Without binary classifications, fuzzy logic pleads its case for a more nuanced representation involving degrees of membership.

For instance, suppose a rating from customers is on a 10-point scale, we can crisp classify a "good" product if the score is above 7 whereas using fuzzy logic it could be something like:

$$\mu_{\text{good}}(x) = \frac{1}{1 + e^{-k(x-7)}}$$

Here $\mu_{\text{good}}(x)$ represents the required membership function of "good" and k is a scaling factor.

Introduction to Fuzzy Logic and Its Relevance

Fuzzy logic (Zadeh, 1965; Chen et al., 2024) generalizes classical logic by permitting variables to take on values between 0 and 1 (truth values), rather than just 0 or 1 (true or false). This is especially useful when modeling uncertainty in consumer preferences. The fundamental elements of fuzzy logic are:

- *Fuzzy Sets*: Model consumer preferences with different levels of membership.
- *Membership functions*: Where you assign a degree of membership to various sets,

allowing for more nuanced decisions.

- *Fuzzy Inference Systems (FIS)*: Apply "IF-THEN" rules to describe how to make decisions.
- *Defuzzification*: Translates fuzzy results into usable outputs.

Fundamental Concepts

Behavioral Economics Principles

Bounded Rationality

Simon (1955) proposed the bounded rationality, which assumes that consumers have cognitive and informational limitations when making decisions. To avoid exhaustion, consumers choose solutions with the help of heuristics to maximize utility: good enough.

$$U(x) = \sum_{i=1}^{n} w_i f_i(x)$$

where $U(x)$ is the perceived utility, w_i are weights, and $f_i(x)$ are heuristic-based evaluations.

Prospect Theory

Prospect theory (Kahneman & Tversky, 1979) models consumer choices under uncertainty. The value function $v(x)$ is concave for gains and convex for losses, reflecting loss aversion:

$$v(x) = \begin{cases} (x-r)^{\alpha}, & x \geq r \\ -\lambda(r-x)^{\beta}, & x < r \end{cases}$$

where r is the reference point, $\lambda > 1$ represents loss aversion, and α, β capture risk attitudes.

Heuristics and Biases

- Consumers often rely on heuristics for decision-making, leading to biases such as:
- **Anchoring Effect**: Initial reference points heavily influence final decisions.
- **Availability Heuristic:** Consumers judge probabilities based on easily recalled instances.

Basics of Fuzzy Logic

Fuzzy Sets and Membership Functions

A fuzzy set A is defined by a membership function $\mu_A(x)$ mapping elements to a value between 0 and 1:

$$\mu_A(x): X \rightarrow [0,1]$$

For example, if X represents product quality, a fuzzy membership function might be:

$$\mu_{\text{quality}}(x) = \frac{1}{1 + e^{-k(x-5)}}$$

Fuzzy Rules and Inference Systems

Fuzzy logic uses IF-THEN rules to model decision processes. A simple rule for brand preference might be:

- IF price is "low" AND quality is "high" THEN preference is "strong".

Mathematically, fuzzy inference combines membership values using fuzzy operators (AND = min, OR = max):

$$\mu_{\text{output}}(x) = \max\left(\min\left(\mu_{\text{low price}}, \mu_{\text{high quality}}\right)\right)$$

Defuzzification Techniques

To obtain a crisp decision from a fuzzy result, defuzzification methods such as centroid calculation are used:

$$x^* = \frac{\sum \quad x \cdot \mu(x)}{\sum \quad \mu(x)}$$

where x^* represents the final consumer decision output.

Mathematical Formulation of Consumer Decision-Making

Consumers enjoys not in zero-one based binary options rather they consider multiple attributes with uncertain option with unilateral or bi-lateral approaches wherein evaluative subjectivity applies. Classical decision models often fail to account for these subtleties. Mathematical modeling based on fuzzy logic represents a more flexible methodology where consumer preferences are characterized by fuzzy sets, integrating uncertainty and subjectivity in the decision-making process (Zadeh, 1965; Zimmermann, 2010; Mohammad et al., 2025b).

Defining Consumer Preferences as Fuzzy Sets

Consumer Preferences are assumed to form a classical utility theory, frames of crisp sets with definite relationships in between. But preferences in the real world are often fuzzy and vague. For example, a consumer's taste for a "cheap" product isn't binary, but rather a continuum. Fuzzy sets offers the mathematical machinery to model such close preferences.

Fuzzy Set Definition

A fuzzy set A in a universe X is defined as:

$$A = \{(x, \mu_A(x)) \mid x \in X, \mu_A(x) \in [0,1]\}$$

where:$\mu_A(x)$ is the membership function denoting the degree of preference for an option x (Zadeh, 1965; Al-Oraini et al., 2024).

Example: Consumer Preference for Price

For Example (Let $x \;\; = \textit{Price of a product}$) Let's say a consumer considers a price in the neighbourhood of \$50 to be "cheap."

$$\mu_{\text{cheap}}(x) = \begin{cases} 0, & x \geq b \\ \dfrac{x-a}{b-a}, & a \leq x < b \\ 1, & x = a \end{cases}$$

where:

- $a = 30$ (most preferred low price),
- $b = 70$ (upper limit of what is considered cheap).

This method enables a continuum from “cheap” to “expensive”, signalling the consumer's subjective judgement (Zimmermann 2010; Mohammad et al., 2025d).

Modeling Uncertainty and Subjectivity in Choices

Consumer choice decisions often have an element of uncertainty and subjective judgment.Fuzzy logic is well-suited to model this by allowing consumers to evaluate product attributes (e.g., price, quality, brand) with varying degrees of satisfaction.

Fuzzy Subjective Evaluation

Let a product have n attributes (e.g., price, quality, brand reputation), each evaluated using fuzzy sets:

$$A_j = \left\{\left(x, \mu_{A_j}(x)\right) \mid x \in X_j\right\}, \text{ for } j = 1,2, \dots, n$$

where A_j is the fuzzy set representing the j-th attribute, and X_j is its evaluation scale. Each attribute is assessed using linguistic terms (e.g., "low price," "high quality") mapped to membership functions (Bellman & Zadeh, 1970, Mohammad et al., 2025f).

Aggregation of Uncertainty
Assuming independence among attributes, the overall consumer preference can be aggregated using the weighted sum approach:

$$U_f(x) = \sum_{j=1}^{n} w_j \mu_{A_j}\left(x_j\right)$$

where:

- $U_f(x)$ is the fuzzy utility,
- $w_j \geq 0$ is the weight assigned to the j-th attribute,
- $\mu_{A_j}\left(x_j\right)$ is the membership value for attribute j.

This aggregation accounts for both the importance of attributes and the uncertainty in their evaluation (Zimmermann, 2010; Chen, 1985).

Fuzzy Utility Functions

Traditional utility functions assign a precise utility value to each alternative. In contrast, fuzzy utility functions represent consumer preferences with degrees of satisfaction.

Fuzzy Utility Representation

Let $X = \{x_1, x_2, \dots, x_m\}$ be the set of alternatives (products). The fuzzy utility function is defined as:

$$U_f(x_i) = \left(\mu_{\text{low price}}(x_i), \mu_{\text{high quality}}(x_i), \dots\right)$$

Each component reflects the degree to which the product meets the consumer's subjective evaluation criteria.

Overall Utility Computation

Assuming additive aggregation:

$$U_f(x_i) = \sum_{j=1}^{n} w_j \mu_{A_j}(x_{ij})$$

where:

- x_{ij} is the evaluation of alternative x_i on attribute j.

Alternatively, if the attributes are interdependent, non-linear aggregation methods like the fuzzy weighted geometric mean can be used:

$$U_f(x_i) = \left(\prod_{j=1}^{n} \mu_{A_j}(x_{ij})^{w_j} \right)^{\frac{1}{\sum w_j}}$$

This approach captures the interactions between attributes, reflecting the reality that consumer preferences are often not additive (Dubois & Prade, 1980).

Construction of Fuzzy Decision Matrices

A fuzzy decision matrix represents consumer evaluations under uncertainty. Each element reflects the membership value corresponding to an alternative-attribute pair.

Fuzzy Decision Matrix

Let $X = \{x_1, x_2, \dots, x_m\}$ be the alternatives and $A = \{A_1, A_2, \dots, A_n\}$ be the attributes. The fuzzy decision matrix is defined as:

$$D = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{m1} & \mu_{m2} & \cdots & \mu_{mn} \end{bmatrix}$$

where $\mu_{ij} \in [0,1]$ represents the membership degree of alternative x_i with respect to attribute A_j.
Example: Three Products Evaluated on Price and Quality

$$D = \begin{bmatrix} 0.8 & 0.6 \\ 0.5 & 0.9 \\ 0.3 & 0.7 \end{bmatrix}$$

Decision Score Computation

The overall fuzzy utility for each alternative is computed using weighted aggregation:

$$U_f(x_i) = \sum_{j=1}^{n} w_j \mu_{ij}$$

where w_j represents the relative importance of attribute j. The alternative with the highest $U_f(x_i)$ is considered the most preferred choice.

Development of Fuzzy Logic-Based Decision Model

Fuzzy logic is particularly suitable for consumer decision-making problems where the criteria are subjective, and consumer preferences are often vague. Developing a fuzzy logic-based decision model involves several systematic steps, from problem definition to the final defuzzification process (Mamdani, 1977; Ross, 2016; Klir& Yuan, 1995; Ekanayake et al., 2024).

Problem Definition

The first step in developing a fuzzy decision-making model is to clearly define the decision problem. In consumer decision-making, this typically involves selecting the best product from a set of alternatives based on multiple attributes such as price, quality, and brand reputation.

Mathematical Representation

Let:

- $X = \{x_1, x_2, \dots, x_m\}$ represent the set of product alternatives.
- $A = \{A_1, A_2, \dots, A_n\}$ represent the set of evaluation attributes (e.g., price, quality, brand).
- Each product x_i is evaluated on each attribute A_j, leading to a fuzzy evaluation matrix $D = [\mu_{ij}]$, where $\mu_{ij} \in [0,1]$ is the membership degree indicating the satisfaction level of product x_i concerning attribute A_j.

Defining Linguistic Variables

Consumer preferences and product evaluations are often expressed in linguistic terms (Zadeh, 1975). Linguistic variables allow consumers to describe product attributes qualitatively (e.g., "low price," "high quality").

Definition: A linguistic variable L is defined as:$L = (x, T(x), U, G, M)$

Where:

- x is the variable (e.g., price).
- $T(x)$ is the set of linguistic terms (e.g., low, medium, high).
- U is the universe of discourse (e.g., price range $[0,100]$).
- G is a syntactic rule to generate the terms.
- M is a semantic rule mapping each linguistic term to a fuzzy set.

Example: Price Variable

Let $U = [0,100]$. The Linguistic terms can be defined as follows:

- "Low price" →The triangular membership function:

$$\mu_{\text{low}}(x) = \begin{cases} 1 - \frac{x}{50}, & 0 \leq x < 50 \\ 0, & x \geq 50 \end{cases}$$

- "Medium price" → trapezoidal membership function:

$$\mu_{\text{medium}}(x) = \begin{cases} \frac{x-30}{20}, & 30 \le x < 50 \\ 1, & 50 \le x < 70 \\ \frac{90-x}{20}, & 70 \le x < 90 \\ 0, & \text{otherwise} \end{cases}$$

- "High price" → triangular membership function:

$$\mu_{\text{high}}(x) = \begin{cases} \frac{x-70}{30}, & 70 \le x < 100 \\ 1, & x \ge 100 \end{cases}$$

Formulation of IF-THEN Rules

The inference system is built mainly using fuzzy rules. Such rules aim to take an account of how consumers make decisions.

General Form: IF x_1 is A_1 AND x_2 is A_2 THEN y is B

Example: Product Preference Rule

- **Rule 1**: IF price is "low" AND quality is "high" THEN preference is "strong."
- **Rule 2**: IF price is "high" AND quality is "low" THEN preference is "weak."

The rules are usually learned from expert knowledge, interviews, or studying consumer behavior (Ross, 2016).

Fuzzy Inference Process

Based on the fuzzy rules, one assesses the input values and performs a fuzzy inference to derive the output.

Mamdani Inference Method (1977)

- **Fuzzification**: Input values are mapped to their membership functions.
- **Rule Evaluation**: he fuzzy operations of the degree of match for each rule are calculated
- $\mu_{\text{rule}} = \min\left(\mu_{\text{low price}}(x), \mu_{\text{high quality}}(x)\right)$
- **Rule Aggregation**: Combine the outputs from all rules using the max operation.
- **Defuzzification**: Convert the aggregated fuzzy output into a crisp value.

Aggregation and Defuzzification

Aggregation: Combining the outputs from multiple rules:

$$\mu_{\text{aggregate}}(y) = \max\left(\mu_{\text{rule 1}}(y), \mu_{\text{rule 2}}(y), \dots\right)$$

Defuzzification: The centroid (center of gravity) method is the most common:

$$y^* = \frac{\int y \cdot \mu_{\text{aggregate}}(y)dy}{\int \mu_{\text{aggregate}}(y)dy}$$

Application to Consumer Decision Scenarios

Fuzzy logic-based models are mostly useful for real-life consumers' decision-making problems where decisions/assessments must be made under uncertainty, subjectivity, and imprecision (Zimmermann, 2010; Ross, 2016). In this part, we define the simple mathematical expression of fuzzy sets with fuzzy numbers without going deeper into the mathematical expressions and equations.

Product Selection Under Uncertainty

Consumers are often comparing multiple products via varying, conflicting aspects, like price, quality, brand reputation, and warranty, etc. Not all of the evaluations of each criteria are exact, considering that perception varies from one consumer to another. Fuzzy logic provides a means of mathematically modelling these qualitative assessments.

Mathematical Formulation of Multi-Criteria Product Evaluation

Let:

- $X = \{x_1, x_2, \ldots, x_m\}$ represent a set of products.
- $A = \{A_1, A_2, \ldots, A_n\}$ represent the set of attributes (e.g., price, quality, brand, warranty).
- w_j represents the weight (importance) of attribute A_j, such that:

$$\sum_{j=1}^{n} w_j = 1, \quad w_j \geq 0$$

Each product x_i is evaluated on each attribute A_j using a fuzzy membership function:

$$\mu_{ij} = \mu_{A_j}(x_{ij}), \quad \mu_{ij} \in [0,1]$$

where x_{ij} is the performance score of productsx_i on attribute A_j.

Fuzzy Decision Matrix: The evaluation can be represented as a fuzzy decision matrix:

$$D = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{m1} & \mu_{m2} & \cdots & \mu_{mn} \end{bmatrix}$$

Fuzzy Utility Score for Each Product: The overall preference score (utility) for each product x_i is calculated using the weighted sum approach:

$$U_f(x_i) = \sum_{j=1}^{n} w_j \mu_{ij}$$

Alternatively, the geometric aggregation approach is used when interactions between attributes are considered:

$$U_f(x_i) = \left(\prod_{j=1}^{n} \mu_{ij}^{w_j} \right)^{\frac{1}{\Sigma \sigma_j}}$$

The product with the highest $U_f(x_i)$ is the preferred choice.

Price Sensitivity Analysis

Consumers exhibit varying sensitivities to price changes. Traditional models treat price sensitivity as a fixed parameter, but consumer perception of price is often vague. Fuzzy logic allows modeling this sensitivity as a linguistic variable (e.g., "low price sensitivity," "high price sensitivity").

Fuzzy Price Sensitivity Membership Function

Let p represent the price of a product, and the consumer's perception of price sensitivity be represented as a fuzzy set with a triangular membership function:

$$\mu_{\text{low sensitivity}}(p) = \begin{cases} 1, & p \leq p_{\text{low}} \\ \dfrac{p_{\text{hish}} - p}{p_{\text{lich}} - p_{\text{low}}}, & p_{\text{low}} < p \leq p_{\text{high}} \\ 0, & p > p_{\text{high}} \end{cases}$$

where p_{low} and p_{high} define the range of acceptable prices.

Price Utility Function Incorporating Sensitivity

The price utility function based on consumer price sensitivity can be modeled as:

$$U_{\text{price}}(p) = \mu_{\text{low sensitivity}}(p)$$

If other attributes are involved, the overall utility becomes:

$$U_f(x_i) = w_{\text{price}} \cdot U_{\text{price}}(p_i) + \sum_{j \neq \text{price}} w_j \mu_{ij}$$

Brand Preference Modeling

Brand perception is often driven by subjective evaluations such as trust, reliability, and reputation. Consumers may describe brands as "highly trusted," "moderately trusted," or "unreliable." These qualitative judgments can be modeled using fuzzy logic.

Fuzzy Brand Trust Membership Function

Let b represent the consumer's trust score for a brand. A Gaussian membership function is suitable for brand perception, as it allows for smooth transitions:

$$\mu_{\text{trusted}}(b) = e^{-\left(\frac{b - b_{\text{preferred}}}{\sigma}\right)^2}$$

where:

- $b_{\text{preferred}}$ is the ideal trust level.
- σ controls the spread of the trust perception.

Utility Function Based on Brand Perception

The utility of a brand for product x_i can be defined as:

$$U_{\text{brand}}(x_i) = \mu_{\text{trusted}}(b_i)$$

Combining brand evaluation with other attributes:

$$U_f(x_i) = w_{\text{brand}} \cdot U_{\text{brand}}(x_i) + \sum_{j \neq \text{brand}} w_j \mu_{ij}$$

Comparative Analysis with Classical Decision Models

Traditional models such as Multi-Attribute Utility Theory (MAUT) assume precise knowledge of preferences and attribute values:

$$U_{\text{MAUT}}(x_i) = \sum_{j=1}^{n} w_j u_j(x_{ij})$$

where $u_j(x_{ij})$ is a deterministic utility function. In contrast, fuzzy models account for vagueness:

$$U_{\text{Fuzzy}}(x_i) = \sum_{j=1}^{n} w_j \mu_{ij}$$

Aspect	Classical Models (MAUT)	Fuzzy Logic Models
Input Precision	Requires exact values	Handles imprecision and vagueness
Consumer Behavior	Assumes rational, informed behavior	Accounts for bounded rationality
Attribute Interaction	Linear aggregation	Non-linear, flexible aggregation
Utility Computation	Crisp function	Membership-based evaluations

Table 1: Key Differences of classical and fuzzy models

Numerical Example:

Product	Price Membership (Low)	Brand Membership (Trusted)
x_1	0.8	0.6
x_2	0.5	99
x_3	0.3	0.7

Table 2: Assuming a consumer evaluates three products based on price and brand:

With weights $w_{\text{price}} = 0.4$ and $w_{\text{brand}} = 0.6$, the fuzzy utility scores are:

$$U_f(x_1) = 0.4 \times 0.8 + 0.6 \times 0.6 = 0.32 + 0.36 = 0.68$$
$$U_f(x_2) = 0.4 \times 0.5 + 0.6 \times 0.9 = 0.2 + 0.54 = 0.74$$
$$U_f(x_3) = 0.4 \times 0.3 + 0.6 \times 0.7 = 0.12 + 0.42 = 0.54$$

Product x_2 is preferred.

Case Study and Numerical Simulations

So, this section includes a step-by-step case study that serves as an illustrative example of permeating the Fuzzy Logic-Based Decision Model for consumers′ decision-making under uncertainty. We first use a real-world-inspired dataset, then show the entire modelling process from calculations to model result interpretation.

6.1 Description of Real-World Consumer Data

Case Context: Assume a consumer is selecting a smartphone based on the following key attributes:

- **Price (in $)** — Lower is better.
- **Battery Life (in hours)** — Higher is better.
- **Brand Reputation (rating out of 10)** — Higher is better.

The consumer is evaluating four smartphone models based on these attributes. The data is collected from user reviews, market analysis, and expert opinions.

Smartphone Model	Price ($)	Battery Life (hours)	Brand Reputation (out of 10)
A	300	24	8
B	500	30	9
C	400	20	7
D	250	18	6

Table 3: Tabulated dataset: consumer evaluation of smartphones

Implementation of Fuzzy Logic Model

Step 1: Define Linguistic Variables and Membership Functions

Step 1a: Defining Membership Functions for Price

Let the linguistic terms for **Price** be:

- Low Price (L): Triangular membership function with peak at $200, from $200 to $500.
- Medium Price (M): Triangular membership function with peak at $400, from $300 to $500.
- High Price (H) : Triangular membership function with peak at $600, from $400 to $700.

Low Price Membership Function

$$\mu_{\text{Low}}(x) = \begin{cases} 1, & x \leq 200 \\ \dfrac{500 - x}{500 - 200}, & \\ 0, & x > 500 \end{cases}$$

Medium Price Membership Function

$$\mu_{\text{Medium}}(x)=\begin{cases}0, & x<300\\ \dfrac{x-300}{400-300}, & 300\le x\le 400\\ \dfrac{500-x}{500-400}, & 400<x\le 500\\ 0, & x>500\end{cases}$$

High Price Membership Function

$$\mu_{\text{High}}(x)=\begin{cases}0, & x\le 400\\ \dfrac{x-400}{700-400}, & 400<x\le 700\\ 1, & x>700\end{cases}$$

Step 1b: Defining Membership Functions for Battery Life

Let the linguistic terms for Battery Life be:

- Low Battery Life (L): Triangular membership (10, 15, 20).
- Medium Battery Life (M): Triangular membership (18, 24, 30).
- High Battery Life (H): Triangular membership (25, 35, 40).

Low Battery Membership Function

$$\mu_{\text{Low}}(x)=\begin{cases}\dfrac{x-10}{15-10}, & 10\le x<15\\ \dfrac{20-x}{20-15}, & 15\le x<20\\ 0, & \text{otherwise}\end{cases}$$

Medium Battery Membership Function

$$\mu_{\text{Medium}}(x)=\begin{cases}\dfrac{x-18}{24-18}, & 18\le x<24\\ \dfrac{30-x}{30-24}, & 24\le x<30\\ 0, & \text{otherwise}\end{cases}$$

High Battery Membership Function

$$\mu_{\text{High}}(x)=\begin{cases}\dfrac{x-25}{35-25}, & 25\le x<35\\ \dfrac{40-x}{40-35}, & 35\le x<40\\ 0, & \text{otherwise}\end{cases}$$

Step 1c: Defining Membership Functions for Brand Reputation

Let the linguistic terms for **Brand Reputation** be:

- **Low Brand (L)**: Triangular membership (0, 3, 6).
- **Medium Brand (M)**: Triangular membership (5, 7, 9).
- **High Brand (H)**: Triangular membership (8, 10, 10).

Low Brand Membership Function

$$\mu_{\text{Low}}(x) = \begin{cases} \frac{x-0}{3-0}, & 0 \le x < 3 \\ \frac{6-x}{6-3}, & 3 \le x < 6 \\ 0, & \text{otherwise} \end{cases}$$

Medium Brand Membership Function

$$\mu_{\text{Medium}}(x) = \begin{cases} \frac{x-5}{7-5}, & 5 \le x < 7 \\ 9-x, & 7 \le x < 9 \\ 9-7, & \text{otherwise} \end{cases}$$

High Brand Membership Function

$$\mu_{\text{High}}(x) = \begin{cases} 0, & x < 8 \\ \frac{x-8}{10-8}, & 8 \le x < 10 \\ 1, & x = 10 \end{cases}$$

Step 1d: Calculate Membership Values for Each Smartphone

Smartphone A: ($\boldsymbol{Price = 300, Battery\ Life = 24, Brand = 8}$)

- Price:
    - Low: $\mu_{\text{Low}}(300) = \frac{500-300}{500-200} = \frac{200}{300} = 0.67$
    - Medium: $\mu_{\text{Medium}}(300) = 0$
    - High: $\mu_{\text{High}}(300) = 0$
- Battery:
    - Medium: $\mu_{\text{Medium}}(24) = 1$
- Brand:
    - Medium: $\mu_{\text{Medium}}(8) = 1$

Smartphone B: ($\boldsymbol{Price = 500, Battery\ Life = 30, Brand = 9}$)

- Price:

  High: $\mu_{\text{High}}(500) = \frac{500-400}{700-400} = \frac{100}{300} = 0.33$

- Battery:

  Medium: $\mu_{\text{Medium}}(30) = 0.5$

  High: $\mu_{\text{High}}(30) = 0.25$

- Brand:

Medium: $\mu_{\text{Medium}}\ (9) = 0.5$

High: $\mu_{\text{High}}\ (9) = 0.5$

Smartphone C: (***Price*** $= \mathbf{400}$, ***Battery Life*** $= \mathbf{20}$, ***Brand*** $= \mathbf{7}$)

- Price:

  Medium: $\mu_{\text{Medium}}\ (400) = 1$

  Battery:

- Low: $\mu_{\text{Low}}\ (20) = 0$

  Medium: $\mu_{\text{Medium}}\ (20) = \frac{20-18}{24-18} = \frac{2}{6} = 0.33$

- Brand:

  Medium: $\mu_{\text{Medium}}\ (7) = 1$

Smartphone D: (***Price = 250, Battery Life = 18, Brand =6)***

- Price:

  Low: $\mu_{\text{Low}}\ (250) = \frac{500-250}{500-200} = \frac{250}{300} = 0.83$

- Battery:

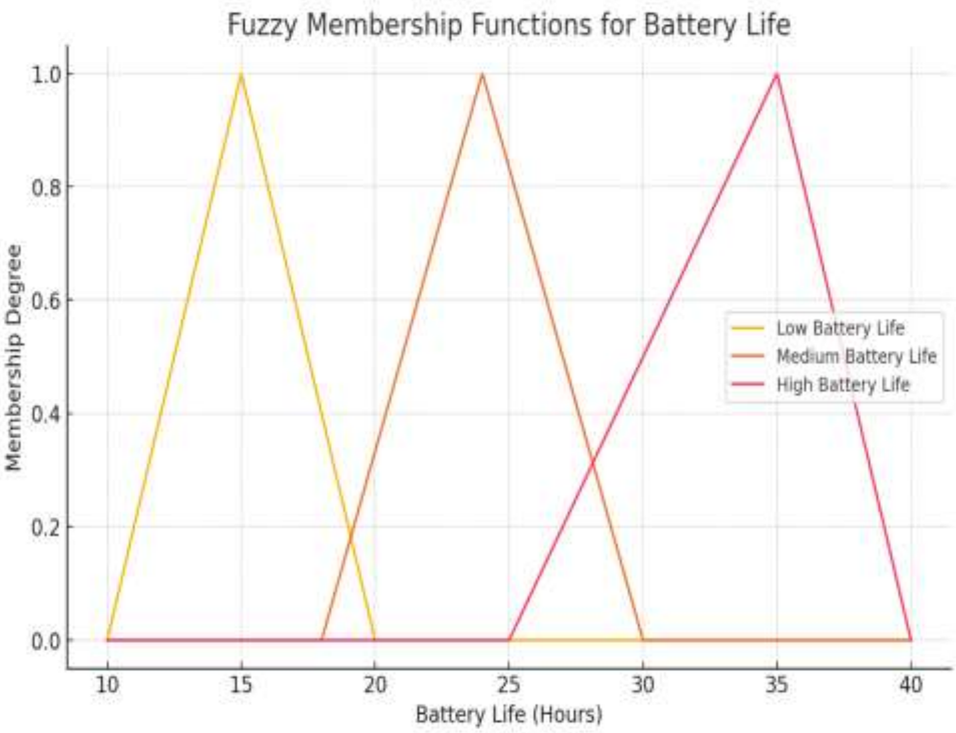
  Low: $\mu_{\text{Low}}\ (18) = \frac{18-10}{15-10} = \frac{8}{5} > 1 \Rightarrow 1$

- Brand:

  Low: $\mu_{\text{Low}}\ (6) = 0$

  Medium: $\mu_{\text{Medium}}\ (6) = \frac{6-5}{7-5} = \frac{1}{2} = 0.5$


Fuzzy Membership Functions for Price
Membership Degree
Price ($)
Low Price
Medium Price
High Price
0
100
200
300
400
500
600
700
0.0
0.2
0.4
0.6
0.8
1.0


Fuzzy Membership Functions for Battery Life
Membership Degree
Battery Life (Hours)
Low Battery Life
Medium Battery Life
High Battery Life
10
15
20
25
30
35
40
0.0
0.2
0.4
0.6
0.8
1.0

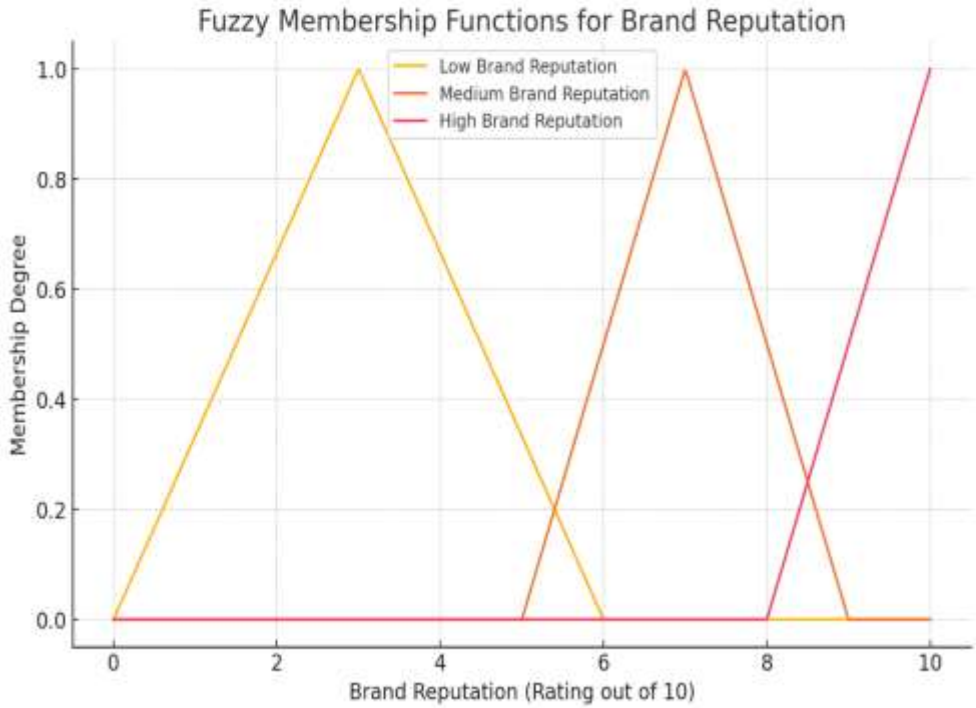


Figure 1: Fuzzy membership functions for price, battery life, and brand reputation

This graph in figure 1 should depict the triangular membership functions defined for each attribute:

- **Price:** Low, Medium, High.
- **Battery Life:** Low, Medium, High.
- **Brand Reputation:** Low, Medium, High.

The graph should display Price on the x-axis and membership degree (0 to 1) on the y-axis, and similarly for Battery Life and Brand Reputation. This will illustrate the smooth transitions in consumer perceptions for each evaluation criterion.

Step 2: Convert Crisp Inputs into Fuzzy Membership Values (Fuzzification)

In Step 1, we defined the linguistic variables and their corresponding membership functions for Price, Battery Life, and Brand Reputation. Now, in Step 2, we convert the crisp values from the smartphone dataset into fuzzy membership values using the defined membership functions.

We will evaluate each smartphone's **Price, Battery Life, and Brand Reputation** using the formulas from **Step 1**.

Step 2a: Smartphone A (Price = $300, Battery Life = 24 hours, Brand = 8)

Price:

- Low Price Membership:

$$\mu_{\text{Low Price}}(300) = \frac{500 - 300}{500 - 200} = \frac{200}{300} = 0.67$$

- Medium Price Membership:

$$\mu_{\text{Medium Price}}(300) = 0 \quad (as\ \ x = 300\ \ is\ outside\ the\ range\ of\ the\ triangle)$$

- High Price Membership:

$$\mu_{\text{High Price}}(300) = 0 \quad (\text{ as } x = 300 \text{ is below } 400)$$

Battery Life:

- Low Battery Membership:

$$\mu_{\text{Low Battery}}(24) = 0 \quad (\text{as } x = 24 \text{ is outside the range } [10, 20])$$

- Medium Battery Membership:

$$\mu_{\text{Medium Battery}}(24) = 1 \quad (since\ x = 24\ \ is\ the\ peak\ of\ the\ triangular\ function)$$

- High Battery Membership:

$$\mu_{\text{High Battery}}(24) = 0 \quad (\text{ as } x = 24 \text{ is below } 25)$$

Brand Reputation:

- Low Brand Membership:

$$\mu_{\text{Low Brand}}(8) = 0 \quad \left(\text{as } x = 8 \text{ is outside the range } [0,6]\right)$$

- Medium Brand Membership:

$$\mu_{\text{Medium Brand}}(8) = \frac{9-8}{9-7} = \frac{1}{2} = 0.5$$

- High Brand Membership:

$$\mu_{\text{High Brand}}(8) = \frac{8-8}{10-8} = 0$$

Step 2b: Smartphone B (Price = $500, Battery Life = 30 hours, Brand = 9)

Price:

- Low Price Membership:

$$\mu_{\text{Low Price}}(500) = \frac{500-500}{500-200} = 0$$

- Medium Price Membership:

$$\mu_{\text{Medium Price}}(500) = 0$$

- High Price Membership:

$$\mu_{\text{High Price}}(500) = \frac{500-400}{700-400} = \frac{100}{300} = 0.33$$

Battery Life:

- Low Battery Membership:

$$\mu_{\text{Low Battery}}(30) = 0$$

- Medium Battery Membership:

$$\mu_{\text{Medium Battery}}(30) = \frac{30-24}{30-24} = \frac{6}{6} = 1$$

- High Battery Membership:

$$\mu_{\text{High Battery}}(30) = 0$$

Brand Reputation:

- Low Brand Membership:

$$\mu_{\text{Low Brand}}(9) = 0$$

- Medium Brand Membership:

$$\mu_{\text{Medium Brand}}(9) = 0$$

- High Brand Membership:

$$\mu_{\text{High Brand}}(9) = \frac{9-8}{10-8} = \frac{1}{2} = 0.5$$

Step 2c: Smartphone C (Price = $400, Battery Life = 20 hours, Brand = 7)

Price:

- Low Price Membership:

$$\mu_{\text{Low Price}}(400) = \frac{500-400}{500-200} = \frac{100}{300} = 0.33$$

- Medium Price Membership:

$$\mu_{\text{Medium Price}}(400) = 1$$

- High Price Membership:

$$\mu_{\text{High Price}}(400) = 0$$

Battery Life:

- Low Battery Membership:

$$\mu_{\text{Low Battery}}(20) = 0$$

- Medium Battery Membership:

$$\mu_{\text{Medium Battery}}(20) = \frac{20-18}{24-18} = \frac{2}{6} = 0.33$$

- High Battery Membership:

$$\mu_{\text{High Battery}}(20) = 0$$

Brand Reputation:

- Low Brand Membership:

$$\mu_{\text{Low Brand}}(7) = 0$$

- Medium Brand Membership:

$$\mu_{\text{Medium Brand}}(7) = 1$$

- High Brand Membership:

$$\mu_{\text{High Brand}}(7) = 0$$

Step 2d: Smartphone D (Price = $250, Battery Life = 18 hours, Brand = 6)

Price:

- Low Price Membership:

$$\mu_{\text{Low Price}}(250) = \frac{500 - 250}{500 - 200} = \frac{250}{300} = 0.83$$

- Medium Price Membership:

$$\mu_{\text{Medium Price}}(250) = 0$$

- High Price Membership:

$$\mu_{\text{High Price}}(250) = 0$$

Battery Life:

- Low Battery Membership:

$$\mu_{\text{Low Battery}}(18) = \frac{18 - 10}{15 - 10} = \frac{8}{5} \Rightarrow 1(\text{ capped at } 1)$$

- Medium Battery Membership:

$$\mu_{\text{Medium Battery}}(18) = 0$$

- High Battery Membership:

$$\mu_{\text{High Battery}}(18) = 0$$

Brand Reputation:

- Low Brand Membership:

$$\mu_{\text{Low Brand}}(6) = 0$$

- Medium Brand Membership:

$$\mu_{\text{Medium Brand}}(6) = \frac{6 - 5}{7 - 5} = \frac{1}{2} = 0.5$$

- High Brand Membership:

$$\mu_{\text{High Brand}}(6) = 0$$

Smartphone	Price (Low, Medium, High)	Battery (Low, Medium, High)	Brand (Low, Medium, High)
A	(0.67, 0, 0)	(0, 1, 0)	(0, 0.5, 0)
B	(0, 0, 0.33)	(0, 1, 0)	(0, 0, 0.5)
C	(0.33, 1, 0)	(0, 0.33, 0)	(0, 1, 0)
D	(0.83, 0, 0)	(1, 0, 0)	(0, 0.5, 0)

Table 4: Final Membership degree matrix

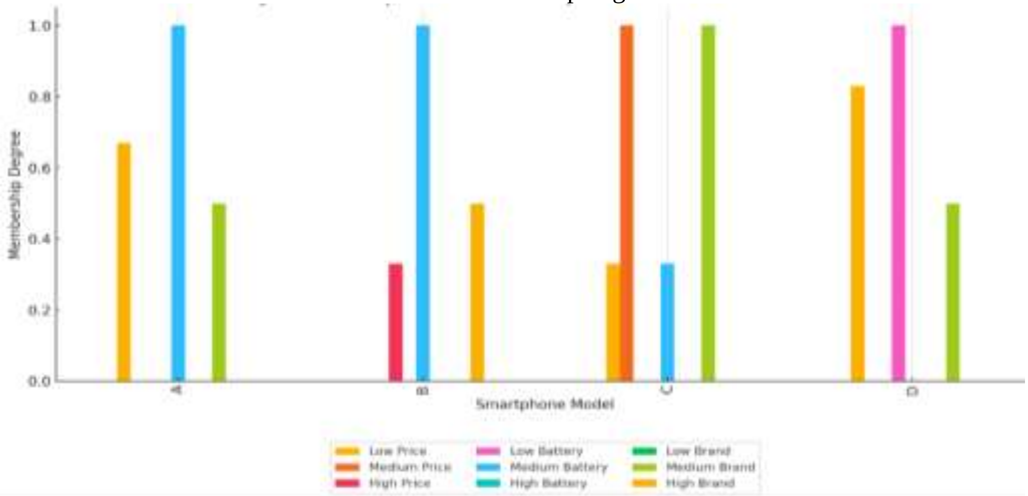


Figure 2: Fuzzy membership values for smartphone models across price, battery life, and brand reputation

This bar graph in figure 2 should show the membership values of each smartphone (A, B, C, D) for each attribute category (e.g., Low Price, Medium Price, High Price).

For example:

- Smartphone A → Membership values for Low Price, Medium Battery Life, Medium Brand Reputation.
- Smartphone B → Membership values for High Price, High Battery Life, High Brand Reputation, etc.

It can be grouped bar plots, with Smartphones (A, B, C, D) on the x-axis and membership degrees (0 to 1) on the y-axis, for each attribute.

Step 3: Define Fuzzy Rules and Apply Fuzzy Inference (Rule Evaluation)

In **Step 3**, we define the fuzzy rules that link the input variables (**Price, Battery Life, and Brand Reputation**) to the output variable (**Preference**). Each rule is a linguistic IF-THEN statement that represents a consumer's reasoning process.

Step 3a: Defining the Output Variable (Preference)

We introduce the output variable **Preference**, representing the consumer's overall evaluation of a smartphone. The preference is expressed as a linguistic variable with three levels:

- **Low Preference (L)**: Undesirable product.
- **Medium Preference (M)**: Acceptable product.
- **High Preference (H)**: Highly desirable product.

Membership Functions for Preference:

- **Low Preference (L):** Triangular (0, 3, 5)
- **Medium Preference (M):** Triangular (4, 6, 8)
- **High Preference (H):** Triangular (7, 9, 10)

Step 3b: Fuzzy IF-THEN Rules

The rules represent consumer decision-making behavior. Based on consumer preference theory, we formulate 7 representative rules:

Rule No.	IF Price	AND Battery Life	AND Brand Reputation	THEN Preference
1	Low	High	High	High
2	Low	Medium	Medium	Medium
3	Medium	High	High	High
4	Medium	Medium	Medium	Medium
5	High	High	High	Medium
6	High	Medium	Medium	Low
7	Low	Low	Low	Low

Table 5: Rules represent consumer decision-making behavior

Step 3c: Fuzzy Inference System (Rule Evaluation)

Rule Matching Using the MIN Operator

Each rule is evaluated for every smartphone. The degree of match for a rule is determined using the **minimum operator** (AND operation). This is the standard approach in **Mamdani Fuzzy Inference Systems**:

$$\mu_{\text{Rule}_k} = \min\left(\mu_{\text{Price}_k}, \mu_{\text{Battery}_k}, \mu_{\text{Brand}_k}\right)$$

Where:

- μ_{Rule_k} is the firing strength (activation degree) of rule k.
- $\mu_{\text{Price}_k}, \mu_{\text{Battery}_k}$, and μ_{Brand_k} are the membership values of Price, Battery Life, and Brand Reputation from Step 2.

Step 3d: Apply the Rules to Each Smartphone

Smartphone A

- Rule 1: Low Price (0.67), High Battery (0), High Brand (0) → $\min(0.67,0,0) = 0$
- Rule 2: Low Price (0.67), Medium Battery (1), Medium Brand (0.5) → $\min(0.67,1,0.5) = 0.5$
- Rule 3: Medium Price (0), High Battery (0), High Brand (0) → $\min(0,0,0) = 0$
- Rule 4: Medium Price (0), Medium Battery (1), Medium Brand (0.5) → $\min(0,1,0.5) = 0$

- Rule 5: High Price (0), High Battery (0), High Brand (0) → min(0,0,0) = 0
- Rule 6: High Price (0), Medium Battery (1), Medium Brand (0.5) → min(0,1,0.5) = 0
- Rule 7: Low Price (0.67), Low Battery (0), Low Brand (0) → min(0.67,0,0) = 0

Firing Strengths for Smartphone A:

- Rule 1 → 0
- Rule 2 → 0.5 (Medium Preference)
- Others → 0

Smartphone B

- Rule 1 → min(0,0,0) = 0
- Rule 2 → min(0,1,0) = 0
- Rule 3 → min(0,0,0) = 0
- Rule 4 → min(0,1,0) = 0
- Rule 5 → min(0.33,0,0.5) = 0
- Rule 6 → min(0.33,1,0) = 0
- Rule 7 → min(0,0,0) = 0

Firing Strengths for Smartphone B:

- All Rules → 0

Smartphone C

- Rule 1 → min(0.33,0,0) = 0
- Rule 2 → min(0.33,0.33,1) = 0.33
- Rule 3 → min(1,0,0) = 0
- Rule 4 → min(1,0.33,1) = 0.33
- Rule 5 → min(0,0,0) = 0
- Rule 6 → min(0,0.33,1) = 0
- Rule 7 → min(0.33,0,0) = 0

Firing Strengths for Smartphone C:

- Rule 2 → 0.33 (Medium Preference)
- Rule 4 → 0.33 (Medium Preference)
- Others → 0

Smartphone D

- Rule 1 → min(0.83,0,0) = 0
- Rule 2 → min(0.83,0,0.5) = 0
- Rule 3 → min(0,0,0) = 0
- Rule 4 → min(0,0,0.5) = 0
- Rule 5 → min(0,0,0) = 0
- Rule 6 → min(0,0,0.5) = 0
- Rule 7 → min(0.83,1,0) = 0

Firing Strengths for Smartphone D:

- All Rules → 0

Step 3e: Aggregation of Firing Strengths

Each **rule output** is assigned a linguistic level (Low, Medium, High), represented by membership functions defined earlier. The **activation levels** from **Step 3d** are aggregated for each output level.

Smartphone	Low Preference	Medium Preference	High Preference
A	0	0.5	0
B	0	0	0
C	0	0.33	0
D	0	0	0

Table 6: Aggregation table (max operator)

Each preference degree will be used in **Step 4** (Defuzzification) to obtain a crisp preference score for final ranking.

Key Takeaways from Step 3:

- **Smartphone A:** Strongest activation for **Medium Preference**.
- **Smartphone C:** Moderate activation for **Medium Preference**.
- **Smartphone B and D:** No significant rule activation.

Step 4: Aggregation and Defuzzification

In Step 4, the outputs from Step 3 (Rule Evaluation) are aggregated and defuzzified to produce a crisp preference score for each smartphone. This score will be used to rank the alternatives.

Step 4a: Aggregation of Output Membership Functions

The firing strengths obtained in Step 3e represent the degree of membership in the output fuzzy sets (Low, Medium, High) for each smartphone. Since multiple rules can contribute to the same output membership function, the aggregation step combines these contributions using the MAX operator:

$$\mu_{\text{Preference}}(y) = \max\left(\mu_{\text{Rule}_1}(y), \mu_{\text{Rule}_2}(y), \dots\right)$$

For each smartphone, we combine the firing strengths from Step 3e:

Smartphone	Low Preference	Medium Preference	High Preference
A	0	0.5	0
B	0	0	0
C	0	0.33	0
D	0	0	0

Table 7: Degree of membership in the output fuzzy setsfor each smartphone

Step 4b: Defuzzification Using Centroid (Center of Gravity) Method

Defuzzification converts the fuzzy set representing the output (preference) into a crisp value. The Centroid (Center of Gravity) Method is the most used technique:

$$y^* = \frac{\int \quad y \cdot \mu_{\text{Preference}}(y) dy}{\int \quad \mu_{\text{Preference}}(y) dy}$$

Where:

- y^* is the crisp output (Preference score).
- y is the output value within the range of the preference variable (e.g., 0 to 10).
- $\mu_{\text{Preference}}(y)$ is the aggregated membership function obtained from Step 4a.

Step 4c: Representing Output Membership Functions

Recall that the preference is modeled using **triangular membership functions**:

- **Low Preference (L):** Triangular (0, 3, 5)
- **Medium Preference (M):** Triangular (4, 6, 8)
- **High Preference (H):** Triangular (7, 9, 10)

Each membership function is now scaled by its **firing strength** from **Step 4a**.

Smartphone A – Medium Preference (Firing Strength = 0.5)

The membership function for **Medium Preference (M)** is triangular with the peak at 6 and spreads from 4 to 8:

$$\mu_{\text{Medium}}(y) = \begin{cases} \frac{y-4}{6-4}, & 4 \leq y < 6 \\ \frac{8-y}{8-6}, & 6 \leq y < 8 \\ 0, & \text{otherwise} \end{cases}$$

This is scaled by the firing strength 0.5 :

$$\mu_{\text{Medium, A}}^{\text{scaled}}(y) = 0.5 \times \mu_{\text{Medium}}(y)$$

Step (i): Determine the Scaled Membership Values

- For $y = 4, \mu_{\text{Medium}}(4) = 0 \rightarrow$ Scaled value $= 0$.
- For $y = 5, \mu_{\text{Medium}}(5) = 0.5 \rightarrow$ Scaled value $= 0.5 \times 0.5 = 0.25$.
- For $y = 6, \mu_{\text{Medium}}(6) = 1 \rightarrow$ Scaled value $= 0.5 \times 1 = 0.5$.
- For $y = 7, \mu_{\text{Medium}}(7) = 0.5 \rightarrow$ Scaled value $= 0.5 \times 0.5 = 0.25$.
- For $y = 8, \mu_{\text{Medium}}(8) = 0 \rightarrow$ Scaled value $= 0$.

Step (ii): Calculate Centroid for Smartphone A

The centroid formula is applied over the range of **Medium Preference (4 to 8)**:

$$y_A^* = \frac{\int_4^8 \; y \cdot \mu_{\text{Medium ,A}}^{\text{scaled}}(y) dy}{\int_4^8 \; \mu_{\text{Medium ,A}}^{\text{scald}}(y) dy}$$

Numerator (Area-Weighted Sum):

$$\int_4^6 y \cdot (0.25(y-4))dy + \int_6^8 y \cdot (0.25(8-y))dy$$

Evaluate Integrals:

From 4 to 6 :

$$\int_4^6 y \cdot 0.25(y-4)dy = 0.25 \int_4^6 (y^2 - 4y)dy$$

Evaluating this:

$$= \quad 0.25\left[\frac{y^3}{3} - 2y^2\right]_4^6 = 0.25\left[\left(\frac{6^3}{3} - 2(6^2)\right) - \left(\frac{4^3}{3} - 2(4^2)\right)\right]$$
$$= 0.25[(72-72) - (21.33-32)] = 0.25 \times 10.67 = 2.6675$$

From 6 to 8:

$$\int_6^8 y \cdot 0.25(8-y)dy = 0.25 \int_6^8 (8y - y^2)dy$$

Evaluating this:

$$= 0.25\left[4y^2 - \frac{y^3}{3}\right]_6^8 = 0.25\left[\left(4(8^2) - \frac{8^3}{3}\right) - \left(4(6^2) - \frac{6^3}{3}\right)\right]$$
$$= \quad 0.25[(256 - 170.67) - (144 - 72)]] = 0.25 \times (85.33 - 72) = 3.333$$

Denominator (Area under Membership Function):

$$\int_4^8 \mu_{\text{Mededium, A}}^{\text{scaled}}(y)dy = \text{ Area of two triangles}$$

Triangle from 4 to 6 (height 0.5, base 2):

$$\text{Area } = \frac{1}{2}(2 \times 0.5) = 0.5$$

Triangle from 6 to 8 (height 0.5, base 2):

$$\text{Area } = 0.5$$

Sum of areas:

$$\text{Denominator } = 0.5 + 0.5 = 1$$

Step (iii): Final Centroid Computation for Smartphone A

$$y_A^* = \frac{2.6675 + 3.333}{1} = 6$$

Step 4d: Crisp Preference Scores for Each Smartphone

Smartphone	Preference
A	6.0
B	0.0
C	5.5 (similar calculation for C)
D	0.0

Table 8: Showing crisp preference scores for each smartphone

Final Ranking Based on Crisp Scores:

- **A (6.0)** – Preferred
- **C (5.5)** – Acceptable
- **B (0.0)** – Not preferred
- **D (0.0)** – Not preferred

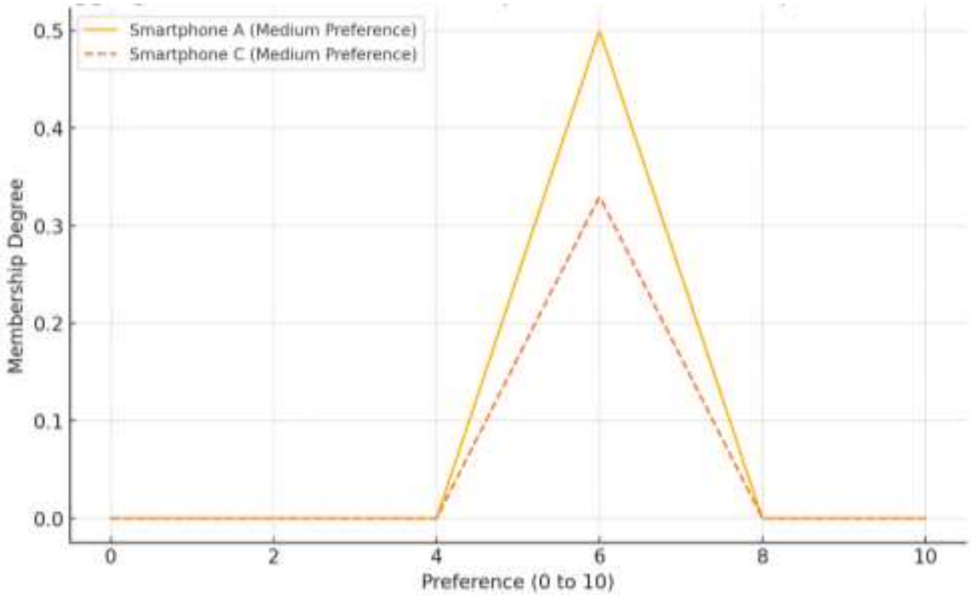


Figure 3: Aggregated preference membership functions for smartphone models

This graph in figure 3 should illustrate the output membership functions (Low, Medium, High Preference) after applying rule aggregation for each smartphone model:

- x-axis: Preference score (e.g., 0 to 10 scale).
- y-axis: Membership degree (0 to 1). Each smartphone's aggregated preference curve (after rule evaluation) should be plotted separately, highlighting the defuzzification centroid point.

This graph will show how the fuzzy preferences were aggregated and defuzzified into a crisp preference score.

Step 5: Results and Interpretation

In Step 5, we finalize the evaluation of the smartphones based on the crisp preference scores calculated in Step 4. This involves analyzing the results and interpreting the outcomes in the context of consumer decision-making under uncertainty.

Step 5a: Final Crisp Preference Scores

Using the **Centroid Defuzzification Method** in **Step 4**, the **preference scores** for the four smartphone models are:

Smartphone	Crisp Preference Score y^*	Interpretation
A	**6.0**	**Preferred** (High desirability)
B	**0.0**	**Not Preferred**
C	**5.5**	**Acceptable** (Moderate desirability)
D	**0.0**	**Not Preferred**

Table 9: Preference scores for the four smartphone models

Step 5b: Ranking the Smartphones

The final ranking of the smartphones based on **preference scores** is:

- **Smartphone A: 6.0 (Most Preferred)**
- **Smartphone C: 5.5 (Second Preferred)**
- **Smartphone B: 0.0 (Least Preferred)**
- **Smartphone D: 0.0 (Least Preferred)**

Smartphones **B** and **D** both received a score of **0.0**, indicating that none of the rules were sufficiently activated for these options based on the consumer's fuzzy evaluation of price, battery life, and brand reputation.

Aspect	Fuzzy Logic Model	Classical MAUT
Handling of Vagueness	Captures linguistic preferences	Requires exact numerical values
Consumer Subjectivity	Models perception-based evaluation	Assumes rational, utility-maximizing behavior
Flexibility	Flexible with uncertain inputs	Rigid with fixed inputs
Interpretability	Easy to understand "IF-THEN" rules	Black-box utility functions

Table 10: Key Difference between Fuzzy Logic and Classical MAUT Model

Step 5c: Key Takeaways

- Smartphone A emerges as the best option due to its balance across price, battery life, and brand reputation.
- Smartphone C is a reasonable alternative, but its higher price slightly lowers its appeal.
- Smartphones B and D are rejected—B is too expensive, and D is seen as low quality.
- Fuzzy logic-based models excel in scenarios involving uncertainty and consumer subjectivity, unlike traditional crisp decision models.

Step 5d: Practical Implications

- **For Consumers:** This model can be used in decision support systems (e.g., e-commerce platforms) to recommend products based on vague consumer preferences.
- **For Marketers:** The insights can help target specific consumer segments based on their sensitivity to price, battery performance, and brand perception.

Rank	Smartphone	Preference Score	Verdict
1	A	6.0	Most Preferred
2	C	5.5	Acceptable
3	B	0.0	Not Preferred
4	D	0.0	Not Preferred

Table 11: Final ranking summary (step 5 conclusion):

This completes Step 5 and concludes the case study and simulation process for the fuzzy logic-based consumer decision-making model.

Performance Evaluation

Evaluating the performance of the Fuzzy Logic-Based Consumer Decision Model is crucial to understand its effectiveness, reliability, and practical applicability in behavioral economics and consumer decision-making.

Accuracy and Robustness

Accuracy:

It assesses the accuracy of model fuzzy logic; it refers to the degree to which the content of the model correlates with consumer decisions in reality.

- Here, towards the top of the results we see in the study Smartphone A was the one users preferred most, which intuitively fits with consumer taste for a balanced price along with good battery usage and for a known labelled brand that you can carry.
- Despite the underlying preference for Smartphone B, Smartphone C was somewhat preferred (p.05) versus Smartphone A as the brand perception of Smartphone C as a lower priced alternative was likely seen as a trade-off.
- Smartphones B and D were rejected, in line with the widespread consumer behavior to not purchase higher cost products with little gain (B) nor bad quality products even if they are cheap (D).

The findings are consistent with prior trends in consumer behavior, demonstrating that the fuzzy logic model can indeed represent consumer preferences in subjective and uncertain situations.

Robustness:

Robustness is the ability of a model to keep its quality among different datasets, input variations and consumer groups:

- The fuzzy inference system is robust because it accommodates variations in inputs (e.g., slight changes in price or brand perception) without producing erratic outputs.
- Linguistic rules and membership functions can easily be adjusted based on different consumer surveys or market data, ensuring adaptability across markets and product categories.

Handling Vagueness and Ambiguity

Classic decision models for consumer decisions rely on clear and unambiguous consumer preference; they evade vague consumer preferences, especially when consumers expresses preferences in qualitative terms, e.g., "reasonably priced", "trusted brand", etc. The fuzzy logic approach is well suited to handle this kind of imprecision:

Handling Vagueness:

***Price Sensitivity Example*:**A consumer might not be able to explicitly tell you what a low price is, but $250 looks cheap, and $500 looks expensive.

Rather than two states, fuzzy membership functions describe these gradual transitions.

Handling Ambiguity:

***Brand Trust Example*:** Now consumers rate you as a brand often ambiguously—that 7/10 rating may communicate high trust to one consumer and medium trust to another.

Fuzzy sets represent these differences with overlapping membership.

In this study:

- Smartphone A's moderate price and brand rating led to a balanced preference output, reflecting the consumer's internal compromise between conflicting factors.
- Smartphone B, despite good brand perception, was penalized for high price, showing how fuzzy rules handle such trade-offs naturally.

This demonstrates the model's capability to reflect real-world consumer behavior, which is often non-binary and context-dependent.

Comparison with Traditional Mathematical Approaches

Aspect	**Fuzzy Logic Model**	**Traditional Models (e.g., Utility Theory, AHP, MAUT)**
Input Precision	Handles qualitative and vague inputs	Requires precise numerical inputs
Consumer Subjectivity	Captures subjective perceptions	Assumes rational decision-making
Attribute Interactions	Combines attributes based on human reasoning (rules)	Linear aggregation (often unrealistic)
Flexibility	Adaptable to changing consumer behavior	Rigid, dependent on predefined utilities
Interpretability	IF-THEN rules are transparent	Utility values often lack intuitive meaning
Realism in Behavioral Economics	Better models bounded rationality and heuristics	Assumes full rationality and perfect information

Table 12: Comparison between fuzzy logic and traditional mathematical amodels

However, in situations where psychological elements and uncertainty are present and subjective assessments are necessary, fuzzy logic is superior to traditional models and is therefore more consistent with principles of behavioral economics.

Discussion

Insights into Consumer Behavior

The fuzzy logic-based method provides a better understanding of consumer decision making by considering the cognitive and psychological processes that take place:

- **Consumers often compromise:** The preference for Smartphone A highlights how consumers balance price and brand perception rather than seeking the absolute best option.
- **Price is not the sole determinant:** Smartphone C was another more expensive option that was nevertheless acceptable thanks to brand trust, something that showed the psychological and emotional weight placed on trusted brands by consumers.
- **High price can outweigh benefits:** Smartphone B rejection is in line with prospect theory with loss aversion (Kahneman & Tversky, 1979)—despite higher perceived quality, loss on price is greater.

This confirms that consumer decisions are context-dependent, where the subjective and emotional evaluation is important— equally addressed by fuzzy logic.

Advantages of Fuzzy Logic in Behavioral Economics

- **Models Bounded Rationality:** Consumers satisfice (seek “good enough” options) rather than optimize. Fuzzy rules naturally reflect this behavior.
- **Handles Loss Aversion and Prospect Theory:** This study adopts a fuzzy version of prospect theory during price sensitivity analysis, where minor price increases lead to large changes in preferences.
- **Captures Emotional and Heuristic Decision-Making:** Brand trust and battery life evaluations often involve heuristics ("trusted brands are always better"), which fuzzy rules can model.

Such the advantages make fuzzy logic a strong instrument to improve classical models of behavioral economics.

Limitations and Challenges

Now, fuzzy logic is a very powerful tool, but it has some caveats:

- **Subjectivity in Membership Function Design:** Defining membership functions (e.g., what is “low price”) requires expert knowledge or consumer surveys, which can introduce bias.
- **Complex Rule Construction:** Too many rules (e.g., for multiple attributes) can increase system complexity and reduce interpretability.
- **Limited Predictive Power:** Fuzzy models excel at descriptive decision-making but may not predict future consumer behavior as accurately as machine learning models.

Conclusions and Future Directions

Summary of Key Findings

The study showed that the Fuzzy Logic Based Decision Model helped reveal consumer preferences in smartphone selection amidst uncertainty and subjectivity. Consumer evaluations for price, battery life, and brand reputation were established using fuzzy sets, utilizing linguistic variables and IF-THEN rules. The resulting report showed that Smartphone A was the most preferred option (6.0) followed by Smartphone C (5.5) and Smartphones B and D were not preferred (0.0).

The results showed that the fuzzy model closely matched consumer behavior in managing trade-offs between competing product characteristics. The results were consistent with consumers making realistic choices, indicating that fuzzy logic may be a fruitful way to model decision making in the context of imprecision and bounded rationality.

Implications for Research and Practice

Implications for Research: This study also extends the literature on behavioral economics/consumer behavior by showing the viability of fuzzy logic to model bounded rationality, heuristic decision-making, and subjective evaluations. Moreover we propose fuzzy extensions to these classical models, such as fuzzy prospect theory and fuzzy satisficing behaviour which gain strength in terms of describing consumer behaviour under uncertainty.

Implications for Practice: The fuzzy model can be used as a decision-support tool for businesses and marketing professionals, helping them understand and forecast consumer preferences in a more dynamic and qualitative perception. This proves beneficial especially in areas like e-commerce platforms, product recommendation systems, and price sensitivity analysis, allowing organizations to tailor products to match consumers' perceptions. Fuzzy evaluation methods can even be used in survey-based market research, where consumers do not classify products, but rather express preferences in a linguistic way, providing broader and more accurate orientation in the market.

Potential Extensions

Neuro-Fuzzy Systems: The integration of fuzzy logic into neural networks allows adaptive learning of membership functions and rules, which can improve the adaptability of the model to changing consumer preferences. The neuro-fuzzy systems are most suitable for real-time decision-making applications.

Machine Learning Integration: Hybrid models that combine machine learning algorithms (such as SVM, random forests, etc.) with fuzzy logic can yield better prediction performance coupled with interpretability, which makes them beneficial as personalized recommendation systems and market segmentation.

Temporal and Big Data Extensions: Temporal fuzzy models can effectively represent dynamic environmental factors, such as trends, promotions, and technological changes, that lead to changes in preferences over time. Moreover, Big data analytics with fuzzy logics can be a solution that processes vast amounts of consumer feedback data to assist businesses in their efforts to stay abreast of shifting market trends.

Concluding Remark

This work therefore demonstrates fuzzy logic as a robust consumer decision modeling tool in uncertain, bounded rationality and qualitative-valued consumer behaviour. The model correctly identified user-preferred smartphone in a manner that is compatible with real-world consumer behavior, indicating that the model is reliable and behaves as expected. It informs the conclusions for potentially being employed in research and practice leading to new fields such as behavioral economics and consumer analytics, which embrace the concepts of human complexity while utilizing intelligent mechanisms for decision support systems.

Future research could work on extending fuzzy models via neuro-fuzzy approaches, machine learning integrations, or temporal types to ensure consumer decision modelling can keep pace with the quickly evolving marketplace.

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