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Hedging Clean/ Dirty Energy with Artificial Intelligence: A Comparison Between DCC and GAS Models

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Abstract

From the existing literature, it is important to note that multivariate GARCH models are widely used to forecast correlation and hedging among different kinds of assets, but without providing a comparison with other types of modelling. This paper compares the forecasting performances of classical DCC-GARCH model with that of the novel multivariate Generalized Autoregressive Score (GAS) model in analyzing the volatilities, correlations and hedging effectiveness of Artificial Intelligence (AI) stock ETF with both Clean and Dirty energies. The estimation results show that the degree of connectedness for the Clean-AI pair is more pronounced than that of Dirty-AI. Furthermore, AI-based assets provide significantly better hedging potential and stronger diversification gains for Dirty energy than for Clean energy. DCC-GARCH model effectively captures the persistent hedging structure in Clean energy portfolios, whereas the GAS framework proves more suitable for Dirty energy due to its ability to track abrupt market adjustments and geopolitical shocks. Overall, our empirical results reveal important practical implications.

Keywords: Forecasting, Volatility, Clean energy, Dirty energy, Artificial Intelligence, DCC-GARCH, GAS, Conditional correlation, Hedging.

Introduction

Economic development has been significantly correlated with the rise in energy use and the emission of greenhouse gas (GHG) (see for example, Acheampong, 2018; Alam et al., 2012; Wang and Wang, 2020) and the development of green energy has the potential to break that correlation, thus promoting sustainable development. In order for development to be considered sustainable, energy must be generated with minimal environmental impact and reduced GHG emissions. Furthermore, Societal development and economic advancement are inextricably linked to energy consumption (Zhang et al., 2022). However, basing on the study of Yan et al., 2022, the global energy supply remains predominantly reliant on fossil fuels, which contributes significantly to environmental degradation and is increasingly becoming a constraint on sustainable economic growth. The evolution of the global energy system is directly related to policies and technologies, redefining energy production, distribution and consumption to meet increased demands for sustainability, security and reliability².

Therefore, the energy sector is currently undergoing a profound transformation, driven by the growing need to improve operational efficiency. A range of technologies is needed for transitioning to green energy and reducing reliance on fossil fuels around the world. Renewable and clean energy sources help mitigate the depletion of finite fossil resources, respond to the

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² For more details, see https://www.iea.org/reports/world-energy-outlook-2025?utm_source=chatgpt.com



rapidly growing global energy demand, and alleviate environmental pressures, Assareh and Ghafouri (2023).

Generally, the energy sector is experiencing a profound transformation, fueled by increasing demands for efficiency, sustainability, and operational reliability, Ahmad et al (2021).

Innovative technologies can present an appealing option for economic, social, and environmental objectives. Artificial Intelligence (AI) has become a central driver of the energy transition, fundamentally altering the processes of energy generation, distribution, and consumption, see Raggad and Bouri (2025). Through, it's capacity to analyze vast datasets and support informed, automated decision-making is transforming energy markets by providing effective responses to some of the sector's most critical challenges. The AI also holds substantial potential for enhancing efficiency and optimization across the energy industry. By leveraging AI technologies, the renewable energy sector can improve operational efficiency, lower costs, and advance the transition toward a more sustainable and environmentally system, Zhang et al., (2022).

Given this context, the objective of this paper is to examine the extent to which AI-related assets can provide effective hedging capabilities for different categories of energy sources, such as Clean and Dirty energies. In line with this objective, the paper makes many contributions. Firstly, to the best of our knowledge, it is the first study to explore the relationship between artificial intelligence assets and multiple segments of the energy market-an issue of growing importance as AI is increasingly expected to shape the dynamics and functioning of modern energy systems. Secondly, this paper examines a novel multivariate Generalized Autoregressive Score (GAS) model and compares its results with those of classical models such as the well-known DCC-GARCH model. Thirdly, while many existing studies use DCC-GARCH models to estimate dynamic correlations and optimal hedge ratios, this current paper compares the optimal hedge ratios obtained from DCC type models with those obtained from GAS. This provides a more complete understanding of how optimal hedge ratios vary between different models, highlighting the complexity of variable interactions, specifically in periods of booms and busts. Finally, our paper conducts a portfolio and risk analysis, highlights the potential for investors to allocate substantial weight to the AI stocks and underlines the importance of adopting an appropriate approach, the one which offers the best forecasting performance. The remainder of the article is designed as follows. Econometric framework is detailed in Section 2. Data and preliminary analysis are presented in section 3. Section 4 discusses the empirical findings. The conclusion and policy implications are presented in section 5.

2. Methodology

In this paper, we apply two different models, notably GAS model of Creal et al., (2013) and DCC-GARCH model (Dynamic Conditional Correlation-Generalized Autoregressive Conditional Heteroskedasticity) of Engle (2002) to model the volatility and conditional correlations dynamics, and also the hedge ratios between energy markets and the Artificial Intelligence (AI) stocks.

2.1. DCC-GARCH model

For a long time, the GARCH models are proved to provide a good prediction especially when the characteristics exhibited by financial time series data were identified.

Starting with r_t representing the energy returns, and included an AR (1) term on the information

set (I_{tca}):

$$r_t = \mu_t + ar_{t-1} + \varepsilon_t$$

μ_t is a ($n \times 1$) vector of conditional expectations of r_t at time t . ε_t is a ($n \times 1$) vector of conditional errors which stores the duration of the innovation for each market ($\varepsilon_t = H_t^{1/2} Z_t$). r_{t-1} is a ($n \times 1$) vector of past (lagged) returns and a is a ($n \times n$) matrix associated with these lagged returns.

Let H_t be the conditional covariance matrix of r_t , and decomposed as follows:

$$H_t = D_t R_t D_t$$

where D_t a ($n \times n$) diagonal matrix of the conditional standard deviations of the residuals.

Note that: $D_t = \text{diag}(h_{1,t}^{1/2}, \dots, h_{n,t}^{1/2})$, where the expression of $h_{i,t}$ is univariate GARCH model, presented the conditional variance:

$$h_{i,t} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1}$$

Let R_t a ($n \times n$) matrix of the conditional correlations with time-varying coefficients, as follow:

$$R_t = \text{diag}(q_{1,t}^{-1/2}, \dots, q_{n,t}^{-1/2}) Q_t \text{diag}(q_{1,t}^{-1/2}, \dots, q_{n,t}^{-1/2})$$

Then, DCC-GARCH (1, 1) model can be expressed as:

$$Q_t = (1 - \theta_1 - \theta_2) \bar{Q} + \theta_1 z_{t-1} z'_{t-1} + \theta_2 Q_{t-1}$$

\bar{Q} represents the unconditional correlation matrix of standardized residuals z_{it} ($z_{it} = \varepsilon_{it}/h_{i,t}^{1/2}$).

The parameters θ_1 and θ_2 are jointly govern the mean-reverting behavior of the correlations. Since their sum is strictly less than unity, this allow that the correlations are time-varying rather than constant across time. Let now considering the conditional correlation estimate ($\rho_{i,j,t}$) as follow:

$$\rho_{i,j,t} = \frac{q_{i,j,t}}{\sqrt{q_{i,i,t} q_{j,j,t}}}$$

2.2. GAS model

For simplicity, we restrict our attention to the GAS(p,q) model with $p=1$ and $q=1$. Let r_t denotes a k -dimensional random vector at time t , characterized by the following conditional distribution:

$$r_t | \mathcal{F}_{t-1} \sim p(r_t; \Psi, \theta_t)$$

Where \mathcal{F}_{t-1} denotes the sigma-algebra generated by the history of the time series up to time t and $P(r_t; \Psi, \theta_t)$ is a density function which is fully characterized by a vector of time-varying parameters θ_t and a set of static parameters Ψ .

In GAS (1,1), the dynamic evolution of the time-varying parameters θ_t is governed by the scaled score of the conditional distribution, along with a first-order autoregressive term, as specified below:

$$\theta_{t+1} = K + A s_t + B \theta_t$$

The coefficients vector K , A , and B are matrices of appropriate dimensions. To improve the computation stability and reduce the computational burden of the GAS model, the coefficient matrices A and B are considered to be diagonal (Ardia et al., 2019). Furthermore, the diagonal elements associated with fixed parameters (such as the mean parameters) are set to zero:

$$A = \text{diag}(0, a_\sigma, a_\rho, a_v)$$

$$B = \text{diag}(0, b_\sigma, b_\rho, b_v),$$

where σ , ρ , and v are respectively the volatility, correlation, and shape parameters. It is important to note that s_t is a scaled score function which is given by:

$$s_t = S_t \nabla_t,$$

where, $\nabla_t = \frac{d \ln p(r_t; \theta_t)}{d \theta_t}$ and $S_t := Z_t(\theta_t)^{-\gamma}$, with $Z_t(\theta_t) := E_{t-1}[\nabla_t \nabla_t^T] = -E_{t-1} \left[\frac{d^2 \ln p(r_t; \theta_t)}{d \theta_t d \theta_t^T} \right]$, where E_{t-1} denotes an expectation with respect to $p(r_t; \Psi, \theta_t)$. Via its choice of the scaling matrix S_t , the GAS model allows for additional flexibility in how the score is used for updating. Note that γ is usually taken value in the set $\{0, 1/2, 1\}$.

According to Creal et al. (2013), the GAS model encompasses the well-known observation-driven GARCH model of Engle (1982) and Bollerslev (1986), the ACD model of Engle and Russel (1998) and the ACI model of Russel (2001), as well as most of the Poisson count models considered by Davis et al. (2003) and Davis et al. (2005). Furthermore, the quantity s_t updates the time-varying parameters from θ_t to θ_{t+1} . This mechanism operates as steepest-ascent algorithm that enhances the local fit of the model given the current parameter values. Essentially, the updating procedure is analogous to the well-known Newton-Raphson algorithm. In practical implementation, time-varying parameters such as the variance and degrees of freedom are subject to constraints, whereas the dynamic evolution of the parameters is specified through a linear structure. Usually, the volatilities of the conditional t-distribution in multivariate GAS (1,1) model are naturally regarded as time-varying parameters. Since the multivariate t-distribution is assumed to govern the conditional distribution in the GAS (1,1) model, the corresponding conditional parameters, such as the location, volatility, correlation, and shape parameters are specified respectively, as follow:

$$(\mu, \sigma^2, \rho, \nu)$$

2.3. Hedging

Using the different returns at different positions, the hedge ratio (γ_t) of a financial portfolio can be expressed as follows:

$$\gamma_t = \frac{R_{S,t} - R_{H,t}}{R_{F,t}}$$

where $R_{H,t}$ presents the return on the hedged portfolio, and expressed as follow :

$$R_{H,t} = R_{S,t} - \gamma_t R_{F,t}$$

Where $R_{S,t}$ is the return on a spot position, while $R_{F,t}$ is the return on a future position.

Note that the lower and the weaker time-varying hedge ratios (i.e., more stable), the more appropriate the safe haven status is.

The anticipated hedge ratios for the upcoming period can be obtained from the estimation of different models and expected for a future period in order to build a hedged portfolio (Kroner and Sultan 1993). Let the variance of the hedged portfolio expressed as follow :

$$var(R_{H,t}/I_{t-1}) = var(R_{S,t}/I_{t-1}) - 2\gamma_t cov(R_{F,t}, R_{S,t}/I_{t-1}) + \gamma_t^2 var(R_{F,t}/I_{t-1})$$

Considering (γ_t^*) as the conditional optimal hedge ratio for the portfolio that combines futures (F) and spot (S) assets, Baillie and Myers (1991), and expressed as follow:

$$\gamma_t^*/I_{t-1} = \frac{cov(R_{F,t}, R_{S,t}/I_{t-1})}{var(R_{F,t}/I_{t-1})}$$

The performance of the optimal hedge ratios gathered from different estimations (GAS and DCC-GAECHE) is measured using the HE index, Ku et al. (2007):

$$HE = \frac{var_{unhedged} - var_{hedged}}{var_{unhedged}}$$

Note that the most appropriate model is the one that surpasses other models in terms of hedging effectiveness (HE), (Chang et al., 2011, 2013).

3. Data description and preliminary analysis

This paper uses daily data for S&P Global Clean Energy index (Clean energy), S&P 500 integrated Oil & Gas index (Dirty energy), and the Global X Robotics & Artificial Intelligence (AI). The underlying index of Clean energy tracks the performance of firms functioning in the global business of clean energy, covering both developed and emerging countries. The dirty energy index reflects the performance of companies engaged in oil and gas exploration, production, refining, and distribution activities. The used AI index focused on tracking investment in firms in developed economies that appear to take advantage of the rising adoption and usage of robotics, artificial intelligence, and smart innovations in diverse sectors. Its major constituents include NVIDIA Corp, Intuitive Surgical Inc, ABB Ltd-Reg, Keyence Corp, SMC Corp, Dynatrace Inc, and Fanuc Corp.

The sampled daily data are available in Yahoo. Finance, collected from September 13, 2016 to November 14, 2025, including, accordingly, 2305 observations considering only working days. All prices are expressed in US dollars and the returns are computed as the first natural logarithm of the close price for each variable.

Figure 1 plots the distribution of the prices and returns over sampled period. Strong disparities are reported in terms of downtrends and uptrends. Low prices are specifically reported for all variables during the Covid-19 pandemic crisis. For instance, while a significant downtrend is reported for Dirty energy prices, in the major, different waves of positive and negative variability over time are detected for the major of the sampled economies with a clear uptrend for clean energy, especially in mid-2020. This may likely provide signs of contagion effects and slow transmission of shocks. Furthermore, the distributions of returns show clear heteroskedastic structures of high or low magnitude. More specifically, Figure 1 shows significant waves of high and low volatilities.

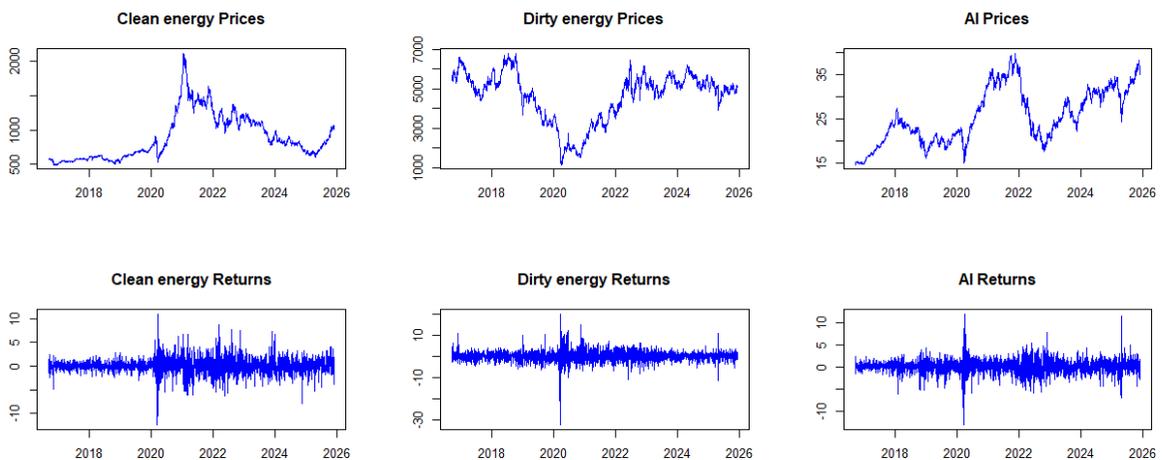


Figure 1: Daily time series plots of prices and returns

The descriptive statistics of the daily returns are reported in Table 1. Based on these statistics, we show a strong difference in the average returns of the sampled indices showing negative average returns in dirty energy and positive average returns for clean energy and AI. As regards the standard deviations, dirty energy presents the higher volatility.

Taken together, the results of the two statistics (Kurtosis and Skewness) provide strong evidence of leptokurtic and asymmetric distribution and supporting the GARCH modeling. Similarly, the results of the Shapiro-Wilk and Jarque-Bera statistics support also the rejection of the null hypothesis of normality of the sampled time series. Moreover, the results of Ljung-BOX and ARCH-LM tests show significant statistical coefficient at the significance level of 1%, suggesting the presence of serial correlation and the presence of an ARCH effect, respectively.

Table 1: Main statistical properties for daily returns

	Clean energy	Dirty energy	AI
min	-12,497	-32,686	-13,246
max	11,033	20,124	11,986
mean	0,024	-0,003	0,038
var	2,775	6,632	2,636
std,dev	1,666	2,575	1,624
coef,var	68,473	-786,001	43,111
skewness	-0,263	-0,698	-0,334
kurtosis	6,326	15,450	7,363
Norm.test (Shapiro-Wilk)	0,935***	0,915***	0,938***
Ljung-BOX (10)	65.668***	22.265***	96.08***
ADF	-11.276***	-11.872***	-12.022***
ARCH-LM (10)	428.1***	347.31***	482.71***

Notes: ADF denotes Augmented Dickey-Fuller unit root tests.

*, **, and *** denote rejection of the null hypothesis at the 10%, 5%, and 1% levels of significance, respectively. The lag length in all the tests has been selected according to the Akaike Information Criteria (AIC).

To check the interdependence between the returns, we calculate the unconditional correlation matrix (Table 2). We find that the correlation is positive for each case, where the strongest and highest correlation is provided between AI and Clean energy.

Table 2: Pearson correlations between daily returns

	Clean energy	Dirty energy	AI
Clean energy	1	0.382	0.592
Dirty energy	0.382	1	0.489
AI	0.592	0.489	1

4. Empirical results and discussion

4.1. Models' specification

In this paper, we compare between the estimation results of the GARCH and GAS models, in order to detect the most suitable model to examine the volatilities, the connection, and the hedging effectiveness among the energy market returns and the AI returns. We use multivariate

GARCH model, notably the DCC-GARCH, and we include an AR (1) term in the mean equation and we specify the univariate residuals by the Student (t) distribution. In fact, the returns showed an auto-correlation (Table 1) and a volatility clustering (Figure 1), which favors estimating this model with non-normality distribution.³

One of the reasons for using the GAS model in our study is the non-normality of the returns series distributions, which is detected in the Table 1 by the simple test of Shapiro-Wilk. But to test the reject rate of normality null hypothesis on sub-samples, we apply the multivariate normality test of Doornik and Hansen (2008) (Doornik-Hansen) for the returns of each G7 stock markets and crude oil. We divide the return series into sub-samples with different sizes of 20, 50, 200 and 500 classified in chronological order, then we test the multivariate normality using the Doornik-Hansen method (at 5% significance level) (see Table 3). The reject rate for each of these sub-samples is calculated by the ratio of rejection times to the total number of fixed sub-samples. The results show that the multivariate normal distribution of our variables is not easy to reject in the case of a small sample and thereafter it is reasonable to use the time varying GAS model with a given conditional distribution.

Table 3: The Doornik-Hansen test results for multivariate normality⁴

	Clean energy-AI				Dirty energy-AI			
Sample size	20	50	200	500	20	50	200	500
Reject rate	0.147	0.297	0.851	1.00	0.133	0.265	0.754	0.997
Total numbers	2285	2255	2105	1805	2285	2255	2105	1805

Note : The reject rate of the null hypotheses of multivariate normality is computed at the 5% significance level.

For simplicity, we only specify the multivariate GAS (1,1) model with a multivariate t-distribution, where the volatilities are naturally considered in this case as time-varying parameters. Nevertheless, it remains to test the variability of the rest of the parameters such as the location, correlation and shape parameters of the conditional t-distribution, using a series of Likelihood Ratio Testing (LRT) approach. We test the null hypothesis $H_0 : M = M_i$ compared to the alternative hypothesis $H_a : M = M_{i+1}$, $i = 0, 1, 2$, and 3, where M_i presents a series of nested model of time-varying parameters, knowing that $M_0 \subset M_1 \subset M_2 \subset M_3$. When $i = 0$, this means that the volatility is the only time-varying parameter, while all the rest of the parameters are time-varying only when $i = 3$. The LRT statistics are asymptotically distributed in chi-squared $\chi^2(k)$ and that the freedom k represents the dimensional difference between the M_i and M_{i+1} models. The results of this test are presented in Table 4 at the 5% significance level. We find that the M_2 model is the best to select for the pair Clean energy-AI, while M_1 is the best for the pair Dirty energy-AI, where the p-value of the LRT in these cases is less than 5%. Then, in what follows, we will consider, for the model GAS (1,1) of Dirty energy and AI, that the parameters of variance and correlation of the conditional t-distribution are time varying, whereas for GAS (1,1) of Clean energy and AI, the parameters of variance, correlation and shape are time varying.

Table 4: GAS model specifications by LRT test

Hypothesis	Clean energy-AI	Dirty energy-AI
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³ For more details see Basher and Sadorsky (2016) and Ahmad et al. (2018).

⁴ Le test de Doornik-Hansen est appliqué pour une combinaison du chaque indice des indices boursiers G7 avec WTI, taux de change et VIX. Le taux de rejet (TR) est calculé au niveau de signification 5%.

	LRT	P.value	Size	LRT	P.value	Size
M0 ↔ M1	2.020	0.364	1304	15.219	0.0005	1304
M1 ↔ M2	11.180	0.004	1304	9.523	0.0085	1304
M2 ↔ M3	4.153	0.386	1304	2.335	0.674	1304

4.2. Regression results

The parameter estimates of the DCC model are presented in Table 5, while those of GAS model are presented in Table 6.

From table 5, we can see that the coefficient μ is statistically significant and positive for all variables. AI presents the highest value of μ , which indicates that this later denotes the most important long-term average variance. The AR (1) term is statistically significant for the used variables, which reinforces the addition of this term in the mean equation. The coefficient of AR (1) is positive for Clean energy and AI index and negative for Dirty energy.

The coefficients α and β are statistically significant for each variable, which constitutes a proof of volatility clustering. All the variables present an evident short-term and long-term persistence, knowing that the short-term one (α) is much lower than the long-term one (β), and their sums are approximately equal to 1, indicating the importance of the long-term persistence for each variable. The estimated coefficients on $a_{e/IA}$ and $b_{e/IA}$ are all positive and statistically significant at the 1% level and their sum is always less than one, which indicates that the dynamic conditional correlations are mean reverting. These results are consistent with the empirical literature that supports the coefficient $a_{e/IA}$ approaches the value of 0 and $b_{e/IA}$ approaches the value of 1, and the sum of two equals a value less than one, as confirmed with Hammoudeh et al. (2010). The Shape parameter (λ) represents the degree of freedom and gives an idea of the shape of the distribution. As more the number of degrees of freedom approaches infinity, more the shape of the student (t) distribution approaches that of a normal. Globally, the DCC-GARCH (1,1) parameters are mainly statistically significant at the 1% level.

Table 5 : DCC model estimates

	Clean energy & AI			Dirty energy & AI		
	coef	S.E	pv	coef	S.E	pv
μ_e	0.050	0.023	0.031	0.037	0.040	0.397
a_e	0.102	0.021	0.000	-0.016	0.0200	0.006
ω_e	0.009	0.004	0.047	0.060	0.033	0.082
α_e	0.061	0.013	0.000	0.055	0.018	0.002
β_e	0.940	0.013	0.000	0.935	0.022	0.000
γ_e	6.498	0.737	0.000	7.434	1.185	0.000
μ_{IA}	0.111	0.024	0.000	0.111	0.024	0.000
a_{IA}	0.009	0.021	0.047	0.009	0.021	0.047
ω_{IA}	0.028	0.011	0.014	0.028	0.011	0.014
α_{IA}	0.101	0.016	0.000	0.101	0.016	0.000
β_{IA}	0.892	0.017	0.000	0.892	0.017	0.000
γ_{IA}	8.384	1.206	0.000	8.384	1.212	0.000

$a_{e/IA}$	0.043	0.008	0.062	0.043	0.010	0.000
$b_{e/IA}$	0.973	0.019	0.000	0.931	0.022	0.000
$\gamma_{e/IA}$	6.970	0.539	0.000	8.855	0.992	0.000
LLK	-7561.197			-8865.826		

Note: The first variable (e) presents the Clean and dirty energies. The second variable (AI) presents the Artificial Intelligence.

With regard to the GAS (1,1) model, Table 6 presents the results of estimating conditional distribution parameters, such as the volatility (σ_e and σ_{AI}), correlation ($\rho_{e/AI}$), and shape (v) parameters. Note that all the time-varying parameters are updated by the scaled score function and their own first order lagged parameters. The According to this table, we find that the estimated parameters of GAS (1,1) model are almost statistical significance at the 1% level.

In line with the estimates obtained from the DCC model, the short-term persistence ($a_{\sigma_e}, a_{\sigma_{AI}}, a_{\rho_{e/AI}}, a_v$), appears markedly lower than its long-term counterpart, as reflected by the relatively large coefficients associated with the coefficients of $b_{\sigma_e}, b_{\sigma_{AI}}, b_{\rho_{e/AI}}$, and b_v . Dirty energy index represents the lowest short-term persistence. These findings indicate that the GAS (1,1) specification captures substantial persistence in both volatility and the nonlinear correlation between the energy indices and the artificial intelligence market. Moreover, the unconditional correlation between Clean energy and AI is higher than that between Dirty energy and AI, which is coherent with the finding in Table 2. Based on log-likelihood criteria, the DCC-GARCH model appears to be more suitable for estimating for the case of Clean energy-AI, contrary to the case of Dirty energy-AI, where the GAS model is preferred.

Table 6 : GAS (1,1) estimates

	Clean energy-AI			Dirty energy-AI		
	coef	S.E	pv	coef	S.E	pv
$K\mu_e$	0.065	0.023	0,000	0.040	0.040	0.155
$K\mu_{AI}$	0.104	0.024	0.000	0.125	0.024	0.000
$K\Phi_e$	0.0002	0.001	0,353	0.004	0.002	0.051
$K\Phi_{AI}$	0.0014	0.001	0,151	0.002	0.001	0.136
$K\rho_{e/AI}$	0.103	0.070	0.070	0.0525	NaN	NaN
K_v	-0.972	0.762	0,100	-2.094	0.215	0.000
a_{σ_e}	0.024	0.004	0,0000	0.025	0.004	0.000
$a_{\sigma_{AI}}$	0.032	0.004	0,0000	0.038	0.005	0.000
$a_{\rho_{e/AI}}$	0.040	0.014	0,003	0.009	0.005	0,031
a_v	5.876	5.692	0,151	—	—	—
b_{σ_e}	0.997	0.003	0,0000	0.994	0.003	0.000
$b_{\sigma_{AI}}$	0.990	0.004	0,0000	0.990	0.004	0.000
$b_{\rho_{e/AI}}$	0.947	NaN	NaN	0.913	0.059	0.000
b_v	0.614	0.274	0,013	—	—	—
LLK	-7590.702			-8852.291		

Note: The first variable (e) presents the Clean and dirty energies. The second variable (AI)

presents the Artificial Intelligence.

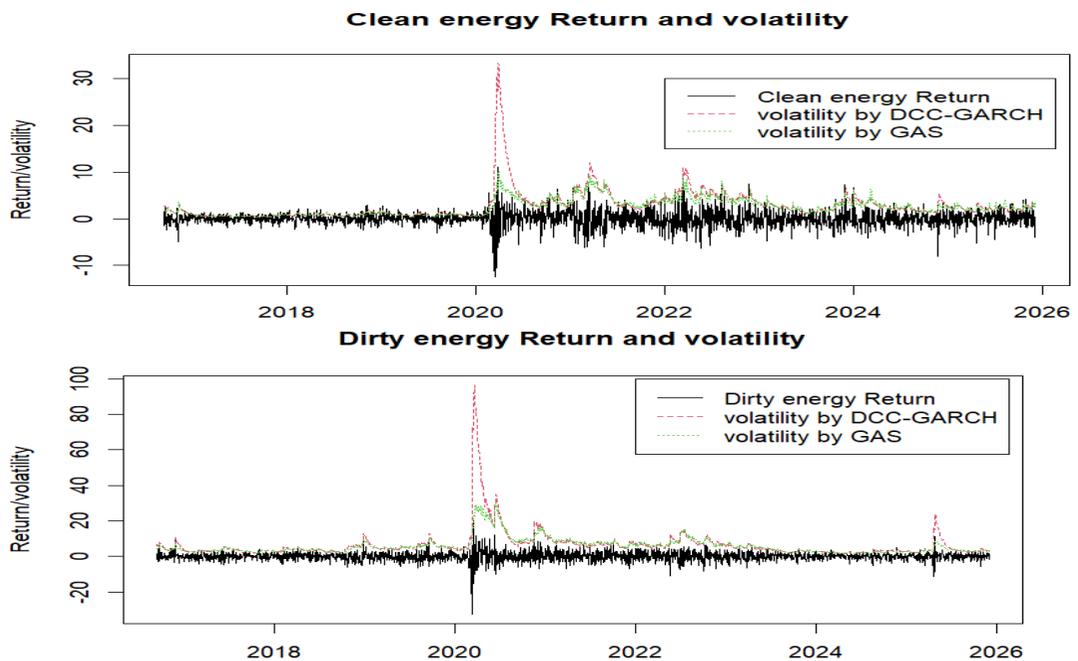
Note : The coefficients a_v and b_v are not existed for the case of Dirty energy-AI, since the Shape parameter in this case is not time-varying.

4.3. Volatilities and dynamic conditional correlations

To study the structure of volatility variation and the nature of the conditional correlations between each type of energy and AI, we plot the curve of two models GAS (1,1) and DCC-GARCH (1,1) in the same figure to make a comparison between the illustrations of two models.

The dynamic volatility and conditional correlations are presented across times, in figures 2 and 3, respectively.

We can notice that the volatility of selected variables corresponds well to the variation of returns series. We show clear waves of high and low volatilities that are easily distinctive as the models capture the time-dependence between shocks and volatility. In addition, we note different magnitude level of conditional volatility for the sampled indices, however the High volatility occurs over the same periods of used data, relating to the Covid-19 crisis (covering 2019-2020). By comparing the illustration of the market volatility of two models, we notice that the DCC-GARCH model provides a very excessive estimate of volatility than that of the GAS model, for some periods of structural return changes, especially during crisis period. This leads to conclude that GAS model is more suitable for detecting the effects of the market volatility persistence.



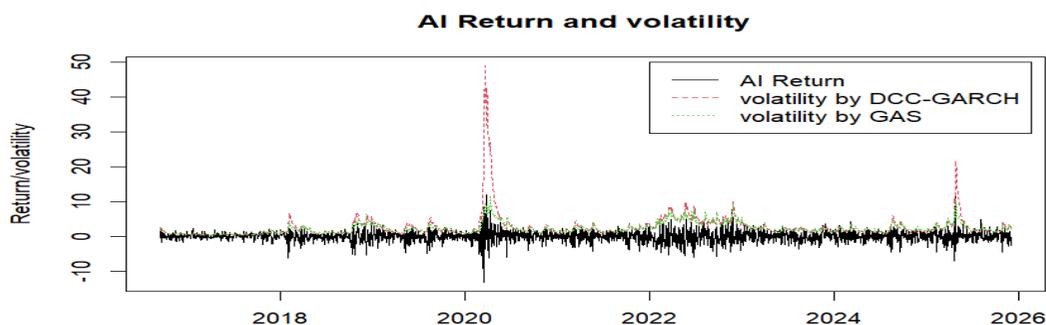


Figure 2: Estimated volatilities

Regarding the dynamic conditional correlation of Clean energy and AI (Figure3), it is persistent, and strictly positive for each model. It is relatively high over the entire study period, with values generally ranging between 0.45 and 0.60, and peaks occasionally reaching 0.70, particularly around 2019-2020. This dynamic suggests that the AI and clean energy markets systematically co-evolve, and reacting similarly to macroeconomic fundamentals and technological development. Thus, clean energy market tends to gain money following increasing digitalization and then AI can act as a technological driver for the clean energy sector and creates a strengthened economic expansion. This is a renewed interest in green investments, reflecting a risk-on market environment, where investors are simultaneously increasing their exposure to innovative sectors (clean energy and AI).

Concerning the case of Dirty energy and AI, the relationship observed is not a methodological artefact; rather, it reflects a genuine and economically meaningful interdependence. Over the full sample period (2016-2025), the dynamic conditional correlations between Dirty energy and AI remain strictly positive across both estimation frameworks, generally fluctuating between 0.2 and 0.6, with peaks occasionally reaching 0.7. These higher correlation episodes, particularly noticeable around 2018-2020 and repeatedly throughout 2023-2025, typically coincide with global risk-on phases, such as the post-Covid-19 recovery, during which both sectors benefit simultaneously from macroeconomic expansions.

Note that during period of turmoil, financial markets tend to exhibit stronger interconnectedness, implying that shocks originating in one sector, such as AI, may propagate more intensively to other sectors including fossil energy. This transmission mechanism contributes to heightened volatility and amplified price co-movements. Moreover, the rapid expansion of cloud computing, GPU-intensive technologies, and large-scale AI models-with their substantial energy requirements-reinforces the dependence of AI-related industries on the energy sector. As a result, increases in AI activity and computational intensity lead to higher operational costs and, consequently, upward pressure on energy prices.

It is also important to note that, the correlation between Dirty energy and AI displays pronounced temporal volatility, characterized by rapid and frequent fluctuations, compared to that of clean energy and AI, which it's more stable. From figure 3, we may confirm that the Clean energy and AI markets seem more closely linked than those of Dirty energy and AI, which it's coherent with Raggad and Bouri (2025).

This behaviour reflects heightened sensitivity to prevailing market conditions in the case of Dirty energy, which indicates that its relationship with AI responds strongly only to short-term shocks, including macroeconomic disturbances, financial uncertainty cycles, technological announcements, and especially variations in oil prices, coordinating with Abdelkader and Si

Mohammed (2025). Finally, for the two cases, both the DCC-GARCH and GAS models produce highly similar trajectories of dynamic conditional correlations, strengthening the credibility of our empirical findings and suggesting a structural coherence in the interdependence dynamics between the two asset classes. While in our empirical setting, the DCC-GARCH appears more appropriate for modelling the dependence between Clean Energy and AI indices, primarily because Clean energy returns exhibit smoother volatility dynamics and more gradual shifts in correlations- conditions under which the DCC framework captures persistence and mean reverting behaviour effectively.

By contrast, the Dirty energy-AI relationship is characterized by sharper regime shifts, stronger reactions to geopolitical and commodity price shocks, and more pronounced tail behaviour. These features generate rapid changes in conditional correlations that the GAS model, with its score-driven updating mechanism, is specifically designed to track in real time. Thus, in this case, the GAS model is more suitable, thanks to its ability in capturing instantaneous variations and modeling the non-linear dynamics of highly volatile markets.

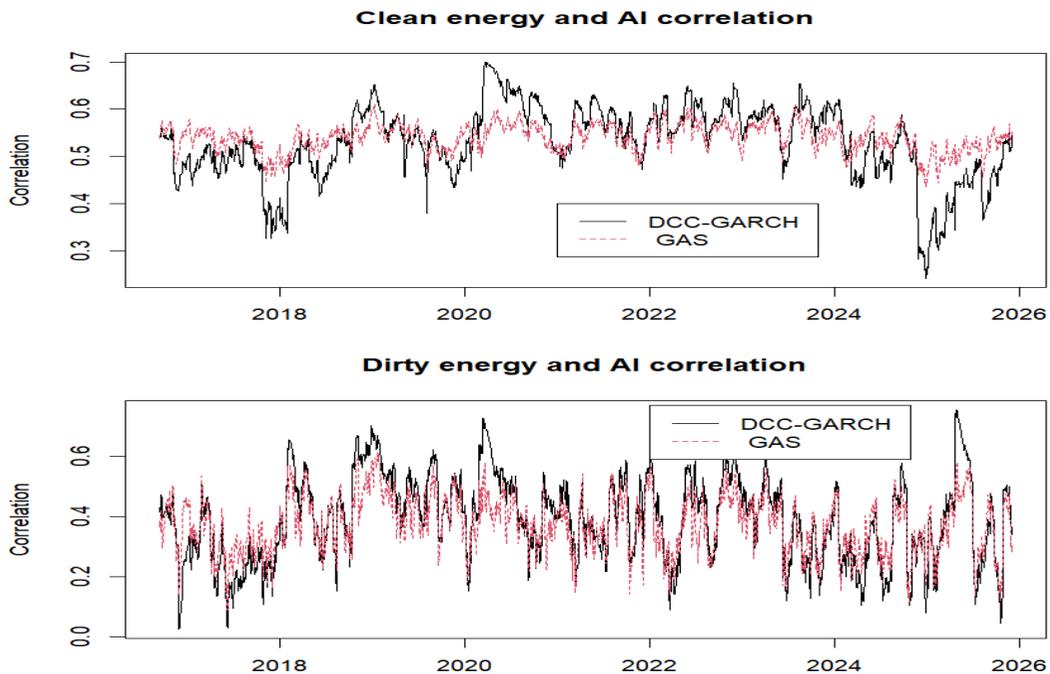


Figure 3: Dynamic conditional correlations

4.4. Forecasting performance

To get the forecast results, we use a rolling window analysis for all 2305 observations. It is an out-of-sample dataset where the models are repeated for each 20 observations of the entire sample. The size of the out-of-sample datasets is defined at 1000 days on the horizon. The forecasting procedure is implemented in a rolling manner by sequentially adding one observation at a time, while keeping the estimation window fixed over the in-sample period. The predictions of the time-varying parameters are generated for multiple horizons ranging from 1 to 1000. We perform a predictive analysis of volatilities and conditional correlations. Based on previous studies, such as that of Klein and Walther (2016), we test and compare the forecast ability of the

mentioned of the two models on the out-of-sample data, by calculating the mean absolute error (MAE) between the volatilities resulting of the different model and the realized volatility (Figure 4), and between the conditional correlation obtained from DCC and GAS models and the realized correlation (Figure 5).

Any deviation between them is considered as the mean absolute error (MAE) (see Engle (2002)):

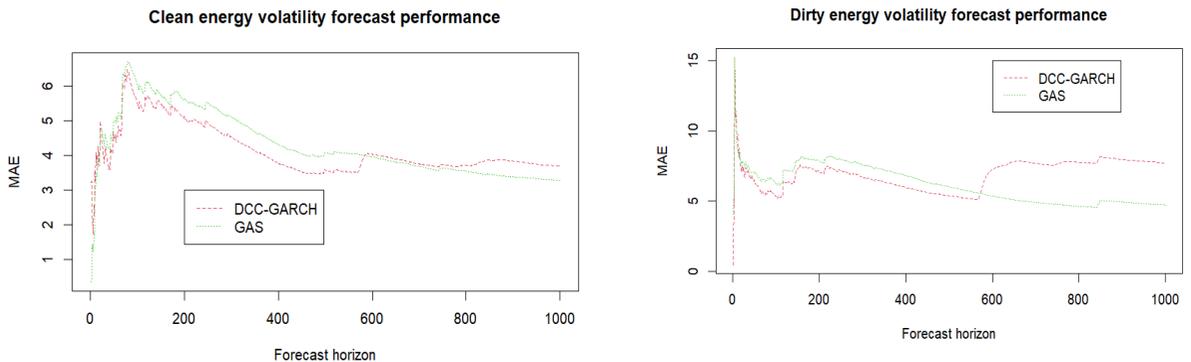
$$MAE_{\sigma^2} = \frac{1}{N} \sum_{t=1}^N |\widehat{\sigma}_t^2 - \sigma_t^2|$$

$$MAE_{\rho} = \frac{1}{N} \sum_{t=1}^N |\widehat{\rho}_t - \rho_t|$$

Where $\widehat{\sigma}_t^2$ and $\widehat{\rho}_t$ respectively represents the volatility and conditional correlation estimated at time t , whereas σ_t^2 and ρ_t respectively denotes the realized volatility and the realized correlation. Further, N presents the size of consecutive sample observations (i.e., the length of horizon days). The lower MAE criteria, the better the forecasting ability gained by the model.

Note to obtain a measure of the realized volatility, we follow Lopez 2001 and Kang et al. 2009, and we employ the returns square of each variable. Whereas, as a proxy measure for realized correlation, we simultaneously multiply cross the two returns sequences, one of the variable to be explained (energy market) and one of the explicative variables (AI index).

The forecasting evaluation shows that the performance of multivariate DCC and GAS models is very limited for forecast horizons of less than 100 days for predicting the volatility of energy returns and less than 200 days for predicting the volatility of AI returns. Whereas, when the forecast horizon is between 100 and 600 days, the DCC-GARCH (1,1) model provides the best forecasting volatility capacity, but beyond 600 days, the multivariate GAS (1,1) model outperforms the DCC-GARCH (1,1) model in forecasting the volatility of selected data. Furthermore, Figure 5 shows that, for the case of AI and Clean energy correlation, the predictive correlation capability of the multivariate DCC-GARCH (1,1) model is mainly superior to that of the GAS (1,1) model over horizon days. However, for the case of Dirty energy and AI relationship, the forecasting results demonstrate that the multivariate DCC-GARCH (1,1) model significantly improves the predictive efficiency of the GAS (1,1) model, particularly for a forecast horizon of less than 600 days. Beyond this horizon, the GAS (1,1) model outperforms the DCC-GARCH (1,1) model. It is therefore important to note that, in most cases, the predictive capability of the GAS and DCC models varies depending on the forecast horizon.



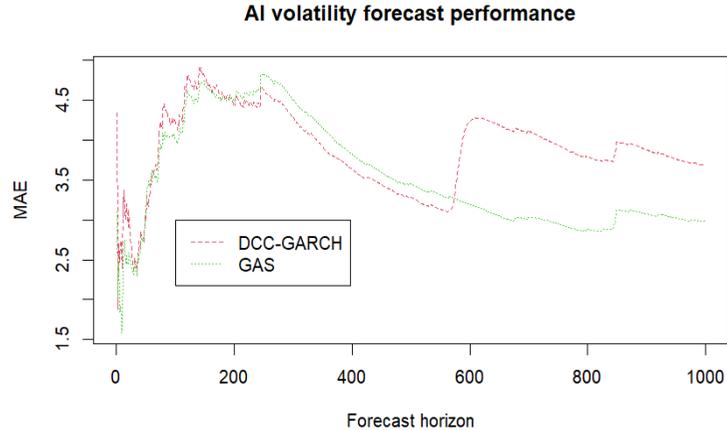


Figure 4 : Forecast performance of volatilities

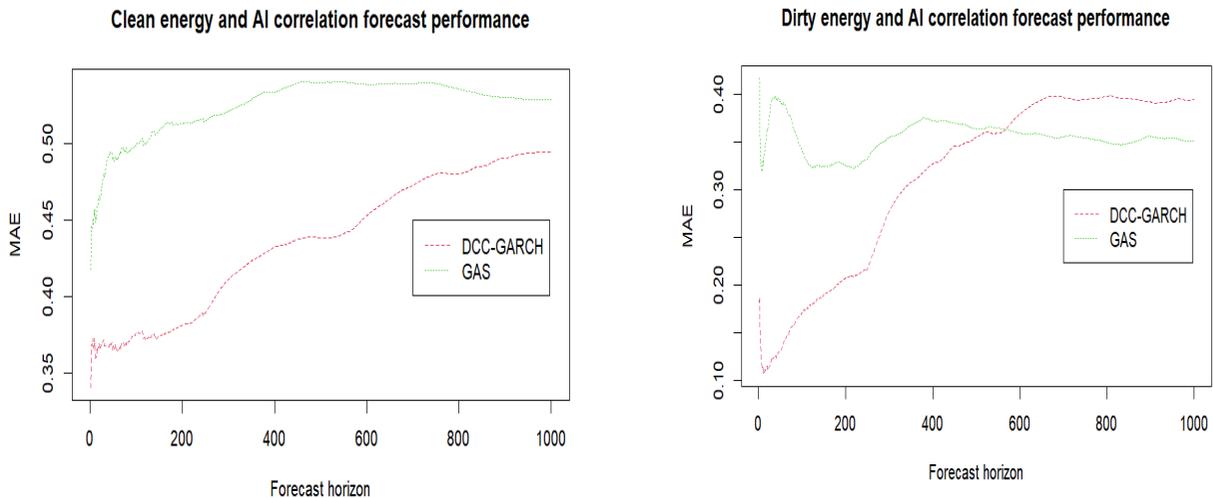


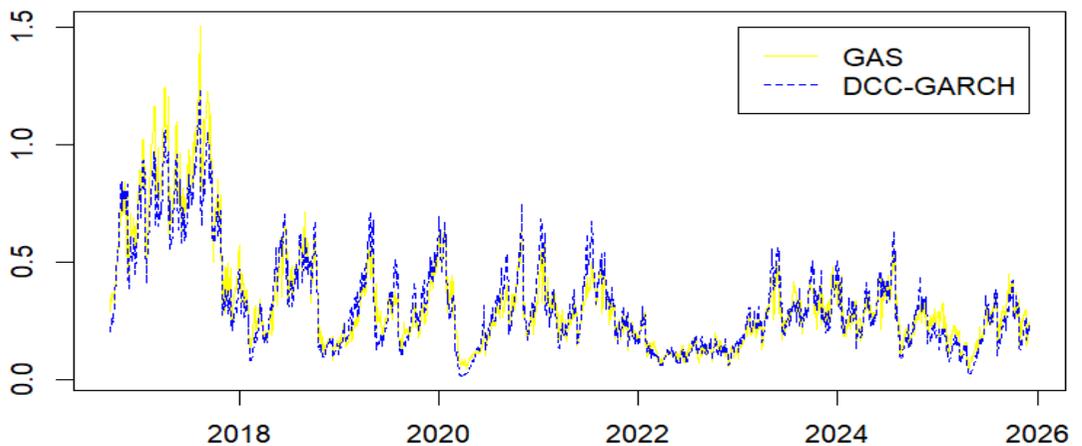
Figure 5 : Forecast performance of conditional correlations.

4.5. Hedging :

The hedge ratio of a financial portfolio can be formulated as a function of the returns associated with different asset positions (Chang et al., 2011; Ku et al., 2007). Forward-looking hedge ratios are obtained from conditional volatility forecasts generated by DCC and GAS models and projected over the investment horizon to construct hedged portfolios (Kroner and Sultan, 1993). Figure 6 illustrate the time-varying optimal hedging ratios (OHR) among the energy markets (clean and dirty energies) and the AI index calculated from GARCH and GAS frameworks under consideration. The hedging ratio measures the extent to which AI index should be included in a portfolio to minimize the risk of energy assets. Higher values imply stronger hedging demand and greater risk transmission, whereas lower values indicate weaker hedging needs and better

Firstly, this Figure shows that, for both cases, the hedging ratios remain structurally positive through the entire period. The upper panel Clean energy-AI exhibits consistently higher hedging ratios than that of Dirty energy-AI, particularly during the early part of the sample (2016-2018), where values frequently exceed 0.8 and even approach 1.2-1.5 under the GAS model. Moreover, in the case of Clean energy-AI hedging, the ratios are less volatile and high persistent across time, indicating that the Clean energy exhibits a structurally high dependence on AI due to deep technological integration and not to determined shocks, which results a weak diversification benefits of AI. In contrast, the dirty energy-AI hedge ratios are more unstable, shock-driven, and weakly integrated, leading to lower hedging costs, high sensitive to short-term shocks and then offering better downside protection during turbulent episodes, consistently with Abdelkader and Si Mohammed (2025). This empirical evidence reveals that AI-based assets provide significantly better hedging potential and stronger diversification gains for Dirty energy than for Clean energy, which it's confirmed with the results found by Raggad and Bouri (2025). In addition, across both figures, we notice that when we calculate the coverage ratios, we find that the models are in greater harmony than when we calculate the correlations. An exception noted during early entire period and crisis episodes, where the GAS model produces more extreme and reactive hedging ratios, reflecting its strong sensitivity to sudden shocks, compared with DCC-GARCH which yields smoother and more stable hedge ratios. This result confirms that, while the DCC-GARCH model effectively captures the persistent hedging structure in Clean energy portfolios, the GAS framework proves more suitable for Dirty energy due to its ability to track abrupt market adjustments and geopolitical shocks.

Clean energy-AI



Dirty energy-AI

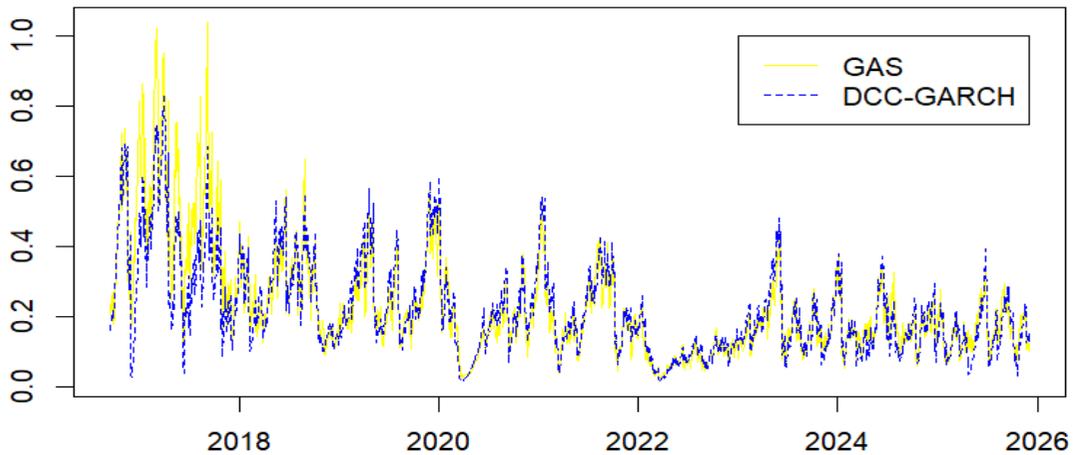


Figure 6 : Time-varying optimal hedge ratios

To further compare the performance of the different OHRs obtained from different models, we calculate the hedging effectiveness (HE) index for each case (see Table 7). Note that a higher HE index indicates a higher hedging effectiveness of the model. Table 7 confirms the observations found in Figure 6.

Table 7 : Hedging effectiveness (HE)

	DCC-GARCH	GAS
Clean energy-AI	0.169	0.156
Dirty energy-AI	0.070	0.083

5. Conclusion

This study investigates the dynamic spillovers in returns and volatility between the AI stock ETF and different segments of the energy market, including clean and dirty energy. The analysis is conducted using daily data spanning from September 13, 2016 to November 14, 2025. This paper mainly employs the novel multivariate GAS(1,1) model to estimate and forecast the volatility, correlation, and hedging between the energy returns and AI returns. Another aim is to compare the power of forecasting the time-varying parameters of multivariate GAS(1,1) model with the classical DCC-GARCH(1,1) model. The empirical findings obtained from the DCC and GAS models are summarized in below.

Firstly, the results of Doornik-Hansen test show that the multivariate GAS(1,1) model is flexible enough to capture the dynamic structure of the conditional distribution of the returns between energies and AI. furthermore, our results reveal that the dependent structure of conditional t-distribution between Clean energy and AI is more complex than those of Dirty energy and AI. This finding is confirmed by basing on the LRT approach which shows that the corresponding shape parameter of the t-distribution have the time-varying characteristic in multivariate GAS (1,1) model of the returns between Clean energy and AI and invariant for the returns of Dirty energy and AI.

Secondly, the correlation between Dirty energy and AI displays pronounced temporal volatility, characterized by rapid and frequent fluctuations, compared to that of clean energy and AI, which

it's more stable. Clean energy and AI markets seem more closely linked than those of Dirty energy and AI. This behaviour reflects heightened sensitivity to prevailing market conditions in the case of Dirty energy, which indicates that its relationship with AI responds strongly only to short-term shocks, including macroeconomic disturbances, financial uncertainty cycles, technological announcements, and especially variations in oil prices.

In addition, it's important to note that the Clean energy exhibits a structurally high dependence on AI due to deep technological integration and not to determined shocks, particularly where the hedging ratios are less volatile and high persistent across time, resulting weak diversification benefits of AI for Dirty energy. Contrary, the dirty energy-AI hedge ratios are more unstable, shock-driven, and weakly integrated, leading to lower hedging costs, high sensitive to short-term shocks and then offering better downside protection during turbulent episodes. This finding divulges that AI-based assets provide significantly better hedging potential and stronger diversification gains for Dirty energy than for Clean energy, which it's coherent with Ragged and Bouri (2025).

Finally, our result affirms that, while the DCC-GARCH model effectively captures the persistent hedging structure in Clean energy portfolios, the GAS framework proves more suitable for Dirty energy due to its ability to track abrupt market adjustments and geopolitical shocks. This finding is also confirmed when estimating the dynamic conditional correlation. This may be explained that, the Dirty energy-AI relationship is characterized by sharper regime shifts, stronger reactions to geopolitical and commodity price shocks, and more pronounced tail behaviour. These features generate rapid changes in conditional correlations that the GAS model, with its score-driven updating mechanism, is specifically designed to track in real time. Thus, in this case, the GAS model is more suitable, thanks to its ability in capturing instantaneous variations and modeling the non-linear dynamics of highly volatile markets. Consequently, DCC provides a more stable and reliable representation of the correlation structure for Clean energy, whereas GAS is better suited to the more abrupt and shock-driven dynamics observed in Dirty Energy markets. These results are confirmed with robustness checks by calculating the MAE and HE indices for both DCC and GAS models.

Our study shows that to optimally hedge the different energies, it is better for investors to consider several models than to rely on a single type. Our findings have important contributions in decision-making for investors, desiring to hedge their energy investments.

In future studies, we can combine these models (DCC and GAS models) with the Wavelet approach to obtain results over different timescales such as in short-term, mid-term and long-term investment horizons, and we identify the robustness of our results.

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