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Design of a Predictive Model Based on Neuromathematics and Machine Learning to Anticipate Academic Performance

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Abstract

The aim of this study was to design a predictive model based on Neuromathematics and Machine Learning to anticipate academic performance in a geometric task. Using the facial recognition software FaceReader, the micro-expressions of 426 students were captured in real time during the construction of a cone in the Cabri Express software. The model achieved 91% accuracy by integrating emotional data and participants' sex as predictive variables. The results indicated that emotions such as happiness and surprise were positively correlated with successful performance, while anger and neutrality were associated with unsuccessful performance. Additionally, significant differences between men and women were identified, highlighting the importance of including sex in the analysis. Neuromathematics allowed the application of knowledge about brain mechanisms in mathematical learning, also guiding pedagogical and didactic processes. Although the study focused on a specific geometric task, its implications are broad, as this approach can be applied to other educational contexts. Among the limitations, the lack of generalization to other mathematical areas is noted, but it is proposed to explore its applicability in future studies in disciplines such as algebra or calculus. This predictive model offers a valuable tool for personalizing teaching, adjusting pedagogical interventions based on emotions and sex, thus improving academic performance and student well-being.

Keywords: *Neuromathematics, Machine Learning, Emotions, Academic Performance, Prediction, Sex.*

Introduction

The term Neuromathematics (Giraldo-Rojas et al., 2021) refers to the application of neuroscience, its knowledge, and advances in brain mechanisms related to learning, particularly in mathematics, as well as in pedagogical and didactic processes aimed at teaching. This science, according to the National Results Report for Colombia - PISA 2018 (Instituto Colombiano para la Evaluación de la Educación, 2020), reveals significantly low performance among students compared to other Latin American countries. This educational challenge has led to a focus on managing emotions and sex differences as crucial aspects of the learning process.

The analysis of sex differences in mathematics learning is crucial, given that research has shown that men and women not only approach mathematics differently but also experience emotions that can affect their performance in different ways (Else-Quest et al., 2010).

Studies have demonstrated that negative emotions towards mathematics directly influence academic performance (Marcos-Merino et al., 2022). In particular, sex differences have been widely documented in the literature on mathematics education, showing that women tend to

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experience higher levels of math anxiety, which can negatively impact their performance (Núñez-Peña et al., 2013; Ganley & Lubienski, 2016). Research such as that by Else-Quest et al. (2010) has found that, although the performance differences between men and women in mathematics may be small, anxiety and the perception of mathematical competence play a crucial role, disproportionately affecting women.

Additionally, Goetz et al. (2013) highlighted that women, in general, experience higher levels of anxiety and stress in math-related areas, which influences their performance compared to their male peers. These findings underscore the importance of including sex differences analysis in studies evaluating academic performance in mathematics.

Performance in mathematics is a critical aspect of students' academic and personal development, and emotions play a key role in this process. Recent studies have shown that negative emotions, such as anxiety and fear, can disproportionately affect women, who tend to report higher levels of math anxiety than men (Núñez-Peña et al., 2013). This emotional difference not only impacts women's confidence in facing math tasks but also directly influences their academic performance (Goetz et al., 2013).

Math anxiety, a specific form of anxiety related to handling mathematical concepts, has been documented as a contributing factor to poor math performance, especially among women (Else-Quest et al., 2010). Moreover, the perception of mathematical competence, which is often lower in women due to social factors and sex-related expectations, also plays a determining role in how students approach math problems (Ganley & Lubienski, 2016). Therefore, studying the impact of emotions on math learning by differentiating by sex is relevant and necessary to fully understand the learning dynamics in educational settings.

From a Neuromathematics perspective, the differences between men and women in emotional and cognitive processing are particularly relevant. Neuroscience research suggests that women show greater activation in the amygdala, a brain region involved in processing emotions such as fear and anxiety, which may affect their ability to concentrate on mathematical tasks under stress (Young et al., 2012). On the other hand, men tend to activate areas related to self-confidence and stress management, which could explain their better performance under high-pressure conditions (Dehaene et al., 2004).

In this context, the present study aimed to analyze sex differences in students' performance in the geometric construction of a cone, using an approach based on facial emotion recognition through the FaceReader software. By including sex as a key variable in the analysis, this study seeks to shed light on how emotional and Neuromathematical differences between men and women can influence their academic performance, in order to develop more personalized and equitable pedagogical interventions.

In the educational field, emotions play a crucial role in the learning process. Giraldo-Rojas et al. (2021) broaden the pedagogy by considering how positive and negative emotions interact to shape teaching strategies and influence the learning process. Ekman (2003) points out that emotions are automatic responses that prepare individuals to face significant events in their lives, triggering changes in the autonomic nervous system and establishing a connection with neuroscience by regulating functions such as breathing and heart rate.

Immordino-Yang and Damasio (2007) suggest that emotions are related to the acquisition of knowledge in the school context. But how do they positively or negatively influence students' academic tasks? According to Ekman and Davidson (1994), basic emotions meet four criteria:

they are learned, arise in similar circumstances, are uniquely expressed, and provoke predictable physiological responses.

Reeve (2010) defines some of these basic emotions within the academic context. Happiness is associated with success and goal achievement, while sadness arises from loss or failure. Anger is triggered when plans are thwarted, and surprise, although brief, precedes other emotions in response to new information. Fear occurs when a threat to well-being is perceived. Disgust, directed toward other people or their actions, differs from personal preferences. In the educational realm, emotions like surprise can help capture students' attention, while boredom can lead to a loss of interest and difficulty in learning (Gómez et al., 2019).

Neuroscience has established a significant connection between emotions and learning. When students associate their efforts with rewards, their motivation increases, as the emotions experienced throughout the school year can influence their academic performance (Marcos-Merino et al., 2022). To identify these emotions, micro-expressions are used—brief facial movements that reveal emotions people try to hide or may not even be aware of (Ekman, 2003). These facial expressions can be crucial for understanding the emotional state of students in an educational setting.

In the pursuit of scientific development and innovation, as well as the improvement of quality of life and optimization of outcomes, Machine Learning (ML) (Cruz et al., 2022) facilitates positive scenarios in classroom teaching processes. These computational algorithms are designed to mimic human intelligence, leading to artificial learning that, among other results, provides suggestions or predictions in specific cases (Rouhiainen, 2018).

In this context, the present project aimed to design and develop a predictive model that, based on Neuroscience, Data Science, and Mathematics Didactics, anticipates students' performance when completing an academic exercise. The research addressed the following question: Can a Neuromathematical study predict a student's performance in constructing a cone using dynamic software? Initially, the research involved a training process with approximately 468 participants, who were introduced to the dynamic software Cabri Express. This software was later used to facilitate the completion of a proposed task, during which participants' facial expressions were recorded simultaneously.

Using the collected data, a ML model was trained to predict the students' performance in solving the mathematical exercise through the proposed software. For this purpose, the facial recognition software FaceReader was implemented, which identifies students' emotions based on micro-expressions manifested during task execution, classifying emotions such as 'Neutral,' 'Happy,' 'Sad,' 'Angry,' 'Surprised,' 'Afraid,' 'Disgusted,' and 'Contempt. **The data from the students' facial micro-expressions were then used to improve the ML model and predict their performance in the task, considering sex as a key variable.**

Finally, a predictive model was established to determine students' performance in constructing a cone through dynamic software. This model was developed using a ML algorithm trained on a collected dataset. Subsequently, a validation process of the predictive model was carried out using a statistical test to determine its accuracy in predicting students' performance with a significant level of precision.

Connection with Neuromathematics

Neuromathematics positions itself as an innovative intersection between neuroscience and mathematics education, providing a comprehensive view of how the brain processes mathematics and the emotions that accompany this learning. Unlike traditional educational approaches, which focus solely on cognitive, pedagogical, or perceptual aspects, Neuromathematics introduces an interdisciplinary perspective that combines the analysis of neural networks involved in mathematical reasoning with emotional regulation.

Mathematics learning involves a series of key brain areas, such as the prefrontal cortex, which is responsible for decision-making and problem-solving, and the parietal cortex, which is related to numerical and spatial manipulation (Dehaene et al., 2004). These areas are connected to the amygdala, which regulates emotions such as fear and anxiety. In the context of mathematical learning, negative emotions can trigger automatic responses that interfere with the activity of the prefrontal cortex, reducing students' ability to focus and solve problems (Young et al., 2012).

This neuro-emotional link is fundamental in Neuromathematics, as it highlights that learning is not just a cognitive process, but one deeply influenced by emotions. Negative emotions, such as fear and anger, can activate the amygdala in ways that inhibit the cognitive activity necessary for effective mathematical reasoning. In contrast, positive emotions, such as joy, can activate the brain's reward system, fostering motivation and improving the student's focus on the mathematical task (Pekrun, 2006).

Emotional Analysis within the Framework of Neuromathematics

The use of tools such as FaceReader, which capture facial micro-expressions, allows for precise real-time data on students' emotional states. Micro-expressions reflect involuntary emotions and provide a direct window into unconscious emotional processing during mathematical tasks (Ekman, 2003). In a Neuromathematical context, these micro-expressions are fundamental as they offer a deeper understanding of how emotions affect the neural networks involved in learning.

This emotional analysis, when combined with Neuromathematics, not only measures the success or failure of a task from a performance perspective, but also considers how emotions are modulating cognitive performance. This approach is key to developing predictive models that capture not only a student's current emotional state but also how this state will influence their ability to successfully complete mathematical tasks.

Differences Compared to Conventional Approaches

Unlike traditional methods, which tend to focus solely on cognitive outcomes or observable behavior (perceptual aspects), Neuromathematics offers an integration between emotional and cognitive processes, using neuroscientific data to enhance the understanding of how students learn mathematics. This approach allows for the creation of more personalized educational interventions, aimed not only at improving academic performance but also at creating an emotionally positive environment that facilitates learning.

The predictive model developed in this study, based on ML, incorporates emotional and neurocognitive data to anticipate academic performance. This provides a powerful tool that goes beyond performance-based analyses, as it allows for predicting how a student might emotionally react to future tasks and how those emotions will influence their performance. This predictive capability not only improves the personalization of learning but can also help identify students at risk of poor performance due to negative emotional states, offering an opportunity for early intervention.

Given these considerations, Neuromathematics provides a holistic approach to analyzing mathematical learning by integrating emotional and neural dimensions. This clearly differentiates it from conventional approaches by providing a solid scientific foundation for the creation of interventions that take into account both the student's emotional state and cognitive ability to solve mathematical problems.

Connection with Previous Research on Facial Recognition and Multimodal Analysis

The use of facial recognition technologies in educational settings has been an active area of research in recent years. Tools like FaceReader, Affectiva, and Noldus Observer have been widely employed to capture and analyze students' emotions during academic tasks, providing valuable insights into how emotions influence performance (Skiendziel et al., 2019; D'Mello & Graesser, 2012).

Previous Research

FaceReader is one of the most widely used technologies for automatic emotion detection, having been validated in multiple studies for identifying basic emotions such as happiness, sadness, anger, surprise, fear, disgust, and contempt (Skiendziel et al., 2019). These studies have shown that negative emotions, such as anxiety and anger, are often correlated with poor academic performance, while positive emotions, such as happiness, are associated with better learning outcomes (Bosch et al., 2016). Similarly, Affectiva and Noldus Observer have enabled real-time emotional analysis during online learning activities, finding that emotions such as surprise or positive engagement tend to predict better academic results (Harley et al., 2015).

In multimodal research, the use of FaceReader has been combined with other biometric devices such as heart rate sensors and electrodermal activity monitors, providing a more comprehensive view of students' emotional and cognitive states (Zhou et al., 2019). These multimodal studies have offered deeper insights into how emotions and attention interact to influence information retention and the resolution of complex problems (D'Mello et al., 2017).

Comparison with the Present Study

While previous studies have used FaceReader to identify the predominant emotions in students and correlate them with academic performance, the novelty of this study lies in the integration of emotions with a Neuromathematical approach, and a predictive model based on ML. Unlike conventional studies that merely analyze the correlation between emotions and performance, this work uses emotional data collected through facial micro-expressions to feed a ML algorithm. This algorithm not only predicts students' performance in mathematical tasks but also does so by taking into account their emotions and sex.

Moreover, this study goes beyond simple emotion detection, examining how these emotions modulate the cognitive processes involved in solving mathematical problems—an area that has been scarcely explored in previous research. While most studies employing FaceReader focus on descriptive correlations, this study adopts a more analytical approach by using these emotions as predictive variables in a ML model designed to anticipate academic performance.

Another significant difference is that this study introduces a temporal dimension to the analysis of emotions by capturing micro-expressions throughout the entire cone-construction task. This allows for observing how emotions change at different stages of the problem-solving process. This temporality is crucial for understanding how immediate emotions (e.g., surprise when facing a difficulty or frustration when making an error) directly influence a student's ability to successfully complete the task.

Innovation in Multimodal Analysis

Additionally, the present study proposes a deeper integration of multimodal analysis by combining facial recognition with the principles of Neuromathematics. The inclusion of emotional data within a Neuromathematical framework not only enables an understanding of how students feel during the task, but also how these emotions impact the neural networks involved in mathematical processing. In this way, the study offers a multidimensional approach that goes beyond traditional analyses, providing a powerful tool for personalizing pedagogical interventions based on students' emotional and cognitive states.

Although facial recognition technologies like FaceReader have already been successfully applied in educational contexts, this study introduces an innovative aspect by combining these technologies with a Neuromathematical approach and a ML-based predictive model. This integration provides a more comprehensive view of how emotions and neural processes intertwine to influence mathematical learning, marking a clear distinction from previous studies.

Methodology

Data Acquisition

Mathematics didactics has recently evolved thanks to a new theoretical approach called **Modeling and Representation with Dynamic Geometry and Conditional Mathematics** (Zabala-Jaramillo et al., 2017), a framework used as the basis for the task consisting of constructing a circular base cone with a slider using the dynamic software Cabri Express.

In this context, a "model" is a structure that replaces the object of study during a simulation conducted by the student and must accurately reflect that object or phenomenon. The use of models in mathematics education is important because it helps students understand abstract concepts in a more concrete and visual way, develop critical thinking and problem-solving skills, and transfer knowledge to different situations.

Representation is responsible for replacing and manipulating the concrete object with the model, making it essential for the model to be effective in the teaching and learning process. Dynamic Geometry focuses on the study of figures and bodies in motion, enriching Modeling and allowing the intuitive exploration of geometric concepts. Finally, Conditional Mathematics is dedicated to the relationships between variables and provides the necessary logical support to understand the behavior of mathematical models and ensure that they accurately represent the properties of the object they substitute.

The theoretical framework that combines Modeling and Representation with Dynamic Geometry and Conditional Mathematics offers a comprehensive and effective approach to mathematics education. This approach emphasizes the importance of constructing and experimenting with models as essential strategies for understanding concepts. According to Zabala-Jaramillo et al. (2017), these concepts interact to generate relationships of validity, particularly between modeling and representation, when a robust connection is established among the elements of the model. This domain is defined through Dynamic Geometry, validated by Conditional Mathematics, allowing the model to operate consistently.

With the clarity of the previous theory, and to complete the task, students initially had to establish three non-collinear points differentiated by a letter to construct a triangle. Then, the perpendicular bisector of each of its sides were added, followed by the search for the circumcenter.

Once the circumcenter was generated, the circle passing through the vertices of the triangle, forming the base, was constructed. This, in turn, allowed for the construction of the cone by linking a slider as its height, automatically generating it above the circle. This phase confirms the correct elaboration through the variation of the slider, noting the change in volume while maintaining the area of the base, which can be moved to demonstrate that the volume does change. This process is illustrated in Figure 1.

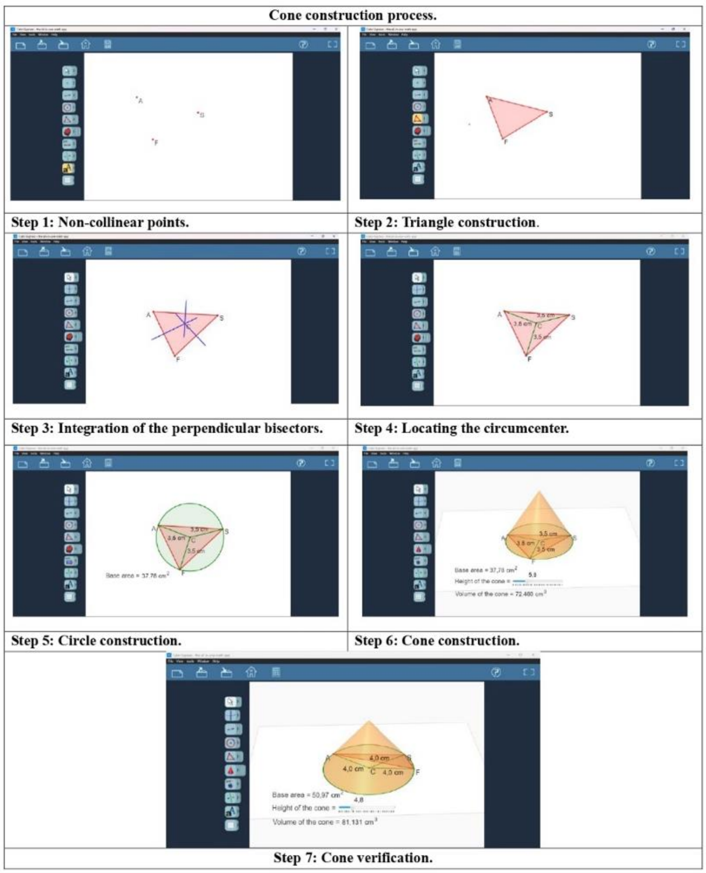


Figure 1. Cone Construction Process. Self-Created (2024).

The task of constructing a cone in a dynamic environment like Cabri Express is an activity carefully selected due to its high cognitive relevance within geometric learning. Geometry is a fundamental part of the mathematics curriculum, and in particular, the visualization and manipulation of three-dimensional objects like the cone are directly related to spatial skills that are crucial for success in this area (Battista, 2007).

When students construct a cone, they must apply key concepts such as non-collinear points, the use of circles, and the variation of parameters like height and radius. These manipulations not only enable a deeper understanding of geometric properties but also facilitate the connection between two-dimensional and three-dimensional geometry, skills that are fundamental for the development of more advanced mathematical thinking.

Spatial reasoning is one of the most important cognitive skills developed through learning geometry and is particularly relevant in solving complex problems (Clements & Sarama, 2011). The ability to visualize and manipulate three-dimensional objects not only aids in geometry but also transfers to other mathematical domains such as calculus, trigonometry, and physics, where students need to understand how objects change and relate in space (Laborde et al., 2006). In fact, studies have shown that a strong capacity for spatial visualization is linked to better performance in abstract and algebraic reasoning tasks, highlighting the importance of tasks like cone construction in overall cognitive development (Battista, 2007).

Furthermore, the use of dynamic geometry software like Cabri Express enhances this learning by allowing students to interact with geometric objects in real-time, visualizing how changes in parameters affect the three-dimensional object. This not only improves students' conceptual understanding but also promotes a more experimental and active approach to learning, which is a key aspect of developing a deep understanding of geometry and other areas related to mathematics (Clements & Sarama, 2011).

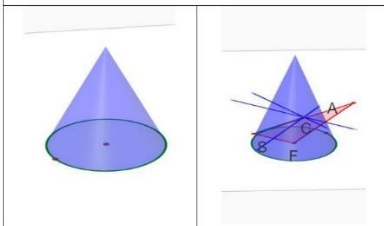
The cone construction task also fosters the interrelation between different areas of mathematics. On one hand, students apply plane geometry concepts, such as constructing triangles and circles, which are essential for defining the base of the cone. On the other hand, an understanding of three-dimensional geometry is required, where students must comprehend how parameters like height and radius interact to affect the volume and area of the base. This interaction promotes the transfer of knowledge between two-dimensional and three-dimensional geometry, skills that are essential for tackling more complex problems in the future, such as calculating areas and volumes in more advanced solid bodies.

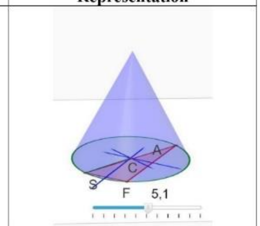
Studies like those by Battista (2007) have shown that the development of spatial visualization through interaction with three-dimensional objects significantly improves geometric understanding. Additionally, research on the use of dynamic geometry software, such as the work of Clements & Sarama (2011), has demonstrated that students not only improve their performance in geometry but also develop greater confidence in tackling complex mathematical problems when they have the opportunity to interact with tools that allow them to experiment with geometric constructions.

In this sense, the task of constructing a cone not only allows students to practice and apply fundamental geometric concepts but also reinforces cognitive skills that are transferable to other areas of mathematics, making it a particularly effective choice for assessing the impact of emotions on mathematical performance.

Constructing a cone using dynamic software is a task with high cognitive relevance and is closely linked to the development of spatial and geometric skills that are essential for success in mathematics. Furthermore, integrating this task into a dynamic software environment facilitates a richer visual and manipulative understanding, promoting active and experimental learning that effectively connects with other mathematical topics. This methodological choice is supported by research that highlights the importance of spatial reasoning in mathematics learning and its positive impact on overall academic performance.

The assessment process and outcome of the task, based on the previously defined theoretical framework of mathematics didactics, determined the construction to be either 'not approved' or 'approved' as presented in Figures 2 and 3, respectively.

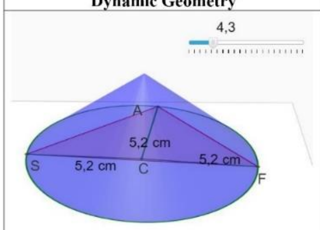


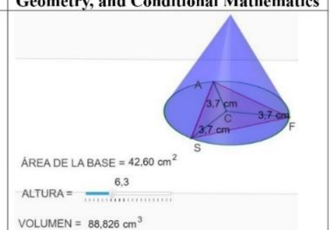


UNAPPROVED PERFORMANCE	
1 – Modeling	**2 – Modeling and Representation**
A model of the cone is created; however, the student does not carry out the construction of the circle using the three non-collinear points, failing to develop the previously established parameters.	The process carried out in the construction of the circle connects the three non-collinear points, but the student did not perform validation or verification.

Figure 2. Description of the Unapproved Design in Cone Construction.

Source: Self-Created (2024).





APPROVED PERFORMANCE	
3 – Modeling, Representation, and Dynamic Geometry	**4 – Modeling, Representation, Dynamic Geometry, and Conditional Mathematics**
In the validation of the cone's consistency, the student indeed utilizes dynamic geometry when, in fact, the three non-collinear points belong to the circumference. Furthermore, by adjusting the height of the cone using the slider, the constructed model demonstrates variations that facilitate the interpretation of properties within the sets of figures and derived bodies.	In addition to validation through dynamic geometry, with conditional mathematics, the student analyzes the variables of the base area and volume that impact the constructed cone model.

Figure 3. Description of the Approved Design in Cone Construction.

Source: Self-Created (2024).

Regarding ethical considerations, Hernández Sampieri et al. (2014) defines the research as transparent, highlighting the definition of the population and the sample selection process for its critique and replication. For the purpose of the project, informed consent was obtained from the parents or guardians of 486 minors who make up the population sample.

During the construction process, students were observed by capturing their facial expressions in videos using Camtasia Studio 8 with a variable duration of 4 to 14 minutes. This approach allowed recording the intensity of emotions and subtle reactions experienced during each phase of the construction. Subsequently, the professional facial coding software FaceReader, developed and validated under the theories of emotions and the Facial Action Coding System (FACS) proposed by Ekman (2004), according to Skiendziel et al., (2019), was implemented. FaceReader encodes the emotions 'Happy', 'Sad', 'Angry', 'Surprised', 'Scared', 'Disgusted', and ‘Contempt’, as well as a ‘Neutral’ expression. (Noldus, 2016).

The FaceReader process consists of three different stages (Skiendziel et al., 2019). Initially, it detects and locates the face by creating an artificial facial model. In this first step, an active template method is implemented, which moves a deformable template of the face over an image, yielding a more probable position of the face. (Yl y Kuilenburg, 2005)

Secondly, it performs a parallel analysis, implementing a classifier based on 500 key points on the face, trained with over 10,000 facial expressions in images that have been manually coded by qualified experts (Zhu, et al., 2023). This model, called the Active Appearance Model, synthesizes the face, describing both the location of these key points and the facial texture, albeit in low dimensionality. (Yl y Kuilenburg, 2005)

Finally, an Artificial Neural Network (ANN) is trained using the appearance vector. With a sufficient amount of data combined with the quality of the facial modeling, facial expressions are classified (Yl y Kuilenburg, 2005). This deep ANN, implemented to recognize patterns, integrates the results of emotion classification during the analysis processes. (Skiendziel et al., 2019)

Thus, the software generates data on the proportion of each of the emotions expressed during any given facial movement (Zhu, et al., 2023) recorded in videos, and directly associates them with an emotion. After detecting the student’s face, the tool projects the microexpressions that, according to Ekman (2003), are those short and extremely rapid movements that individuals manifest even when they are not aware of it, but are projected in their facial action and allow the identification of an emotion. Finally, the software classifies these emotions according to intensity, providing quantifiable data.

According to Noldus (2016), the facial analysis model works by analyzing the characteristics of a human face, such as the shape of the eyes, mouth, and nose, as well as the position of facial muscles. The model uses these characteristics to identify patterns associated with different emotions. Once trained, it can be used to identify emotions in people’s faces. Among other qualities, it has a wide range of applications, in this case for the scientific development of a predictive model. FaceReader stores the data corresponding to people’s emotions in the following process:

- Data are stored in a database of faces. The database contains images of faces from people all around the world.
- The images are labeled with the emotion displayed on the face.
- The database is used to train a ML model.
- The ML model is used to identify emotions on faces.
- The model can be used in a variety of applications.

Similarly, FaceReader analyzes emotions within a 30-millisecond period. During this interval, the platform examines changes in facial features exhibited by individuals to determine the expressed emotion (Noldus, 2016). These changes can include:

- Eyebrow position.
- Eye position.

- Mouth position.
- Facial muscle position.

The 30-millisecond time interval strikes a balance between precision and performance. A shorter interval might not capture all the facial changes associated with an emotion, while a longer one could reduce the software's effectiveness. The facial expressions categorized by FaceReader are presented in graphs and exported as a log file. Each expression is rated on a scale from 0 to 1 according to its intensity, where '0' indicates absence and '1' indicates full presence (Noldus, 2016). The text file contains the video analysis results, including the emotions expressed and the facial characteristics of the registered person, as detailed in Figure 4.

```
Video analysis detailed log

Face Model:General
Calibration:     None
Start time:08/10/2022 16:11:18.750
Filename:  D:\andrés montoya_Camera_Stream.wmv
Frame rate:15.1515151515152

Video Time Neutral     Happy Sad   Angry Surprised  Scared     Disgusted
     Contempt    Valence     Arousal     Gender     Age   Beard Moustache
     Glasses     Ethnicity   Y - Head Orientation  X - Head Orientation
     Z - Head Orientation    Landmarks   Quality    Mouth Left Eye    Right
Eye  Left Eyebrow      Right Eyebrow     Identity
00:00:00.000     0.3220375000.3409702000.0001672180.0000019110.759172400
     0.0000040090.0005451940.2161874000.3404250000.952508100Female
     10 - 20     None  None  No    Caucasian  -10.815640000    -
0.579329400-5.785682000
     153.8783,97.65547,157.4756,92.79959,162.3518,90.33793,166.4991,90.2
5291,170.7005,91.08155,186.2739,89.87095,189.7497,88.41671,194.2888,87.92
598,198.9109,89.63394,202.2059,93.77906,160.5309,104.8454,170.3436,103.54
22,186.7423,102.4626,196.6048,101.9366,176.9492,113.1773,179.6803,108.642
2,182.9952,112.5215,174.3569,127.0427,176.2658,126.7355,179.9755,124.8346
,182.0178,124.7292,184.0835,124.2399,188.4775,125.2018,190.7363,124.9796,
173.8023,127.613,175.9938,127.4229,178.1096,126.8548,180.4425,126.2724,18
2.1823,126.1221,183.9768,125.7393,186.4939,125.7295,188.963,125.8078,191.
3302,125.4482,172.9006,127.5364,175.8875,125.3311,179.1876,122.1113,181.6
408,121.918,184.0643,121.4775,188.3491,123.845,192.196,125.1967,173.5551,
128.4349,174.7607,130.6694,177.5968,131.4112,180.56,131.36,183.0455,131.4
151,185.7576,130.6692,188.312,129.9926,190.8543,128.7287,191.8613,126.176
4    0.860689300Open  Open  Open  Neutral    Lowered    NO IDENTIFICATION
00:00:00.066     0.2769580000.4411520000.0001389910.0000086290.671987800
     0.0000187880.0004762530.1923546000.4406758000.953313400Female
     10 - 20     None  None  No    Caucasian  -9.326829000     0.596965300
     -5.114943000
```

Figure 4. Video Analysis Detailed Log. Source: Facereader Output.

In the first line of the file, the exact moment when the facial expression was captured is recorded. The following lines contain values that indicate the intensity with which the eight basic emotions were detected in the person appearing in the video. In other words, the intensity of each individual facial expression is analyzed and quantified.

According to a classification detailed by Ekman (2003), 'Sad', 'Angry', 'Scared', 'Disgusted', and 'Contempt' are considered negative emotions, while 'Happy' is the only positive emotion. 'Surprised', on the other hand, is neither categorized as positive nor negative. This is identified in the next line as 'Valence', which determines emotions in these two categories. To calculate 'Valence', as per Noldus (2016), the difference in intensity between the 'Happy' expression and the most intense negative expression is computed. The following four lines detail information about the person.

In line with the above, the study uses an approach based on the collection of data related to detected emotions, sex, and performance from the cone construction task, utilizing facial analysis tools and the dynamic geometry software Cabri Express, while the detected emotions, such as happiness, sadness, anger, rage, fear, surprise, and neutrality, were recorded through the

FaceReader software.

Data Processing

Data processing is fundamental for extracting knowledge and making informed decisions. This process includes the acquisition, organization, analysis, transformation, and presentation of data, providing the foundation for understanding information and applying it in decision-making. In this study, data processing began with the collection of facial videos from students, analyzed using the FaceReader software, which provided detailed information on emotions such as happiness, sadness, surprise, fear, and disgust, all relevant to the analysis.

These data were stored in a central database to ensure their integrity throughout the research cycle. Subsequently, a data cleaning and preprocessing phase was carried out to eliminate errors and prepare the information for analysis. Using ML techniques, patterns and trends were uncovered, resulting in valuable insights. The results were presented through charts and tables that facilitated interpretation and supported decision-making.

In addition to emotions, students were classified according to their sex (man or woman), incorporating this variable into the analysis due to evidence of significant differences in emotional management and academic performance between males and females (Núñez-Peña et al., 2013). These differences were included to better understand how emotions and sex affect students' mathematical performance.

As students performed the geometric task, emotional data and sex were linked to their academic performance according to the Theoretical Framework of Mathematics Didactics: *Modeling and Representation with Dynamic Geometry and Conditional Mathematics.* This analysis, based on Neuromathematical principles, explored the relationship between emotions and the activation of brain areas such as the prefrontal cortex and amygdala, which influence students' ability to solve mathematical problems (Young et al., 2012). The study not only measured success in the task but also analyzed how emotions modulate cognitive activity during the problem-solving process.

Finally, the detected emotional data and sex were input into a predictive model based on ANN, integrating principles of neuroscience and ML. This model allowed for the prediction of students' approval or unapproval in the task of constructing a cone, exploring how emotions and sex influence the brain mechanisms associated with learning mathematics. The combination of emotional, neurological, demographic, and ML data provided deeper insights into how students' emotional states and sex impact their academic performance.

The predictive model developed in this study included sex as an explanatory variable along with the emotions captured by FaceReader. The ML algorithm, based on an ANN, was trained to predict the success or failure of students in the geometric task, using both emotions and sex as predictive variables. In this way, the model not only considered the student's emotional state but also how sex influenced academic performance.

Formatting

Data processing in this research begins with the acquisition of information in TXT format extracted from FaceReader records using Python and the Pandas library. Python was chosen for its simplicity and ability to handle large amounts of data, while Pandas is essential for converting the data into a structured format using DataFrames, which organize the information into rows and columns for easy analysis and manipulation.

The process begins by reading and parsing TXT files using Python's operating system library, filtering by the .txt extension, and storing each line in a list. Then the data is organized into DataFrames, with columns appropriately labeled, allowing for efficient handling of the information. The processed data are stored in a dictionary using cleaned filenames as keys, ensuring efficient access and manipulation. The choice of tools such as Python and Pandas is crucial for the research, ensuring efficiency and accuracy in extracting knowledge from the data.

Data Cleaning

Data cleaning is an essential and challenging process in data management and analysis. It includes error identification and correction, outlier detection, missing value handling, data normalization, deduplication, and validation. This process is critical to achieving accurate and reliable results in both data analysis and ML model building, ensuring informed decisions and the extraction of meaningful insights.

To ensure that the data was in a proper state prior to storage, a cleaning code was implemented. A key step was to remove rows with None or NaN values using data = data.dropna(), which ensures that only DataFrames without numerical noise are stored, contributing to data integrity. In addition, additional lines of code were implemented such as data.drop_duplicates() to remove duplicate rows and another to remove rows with a value of "FITFAILED" to maintain consistency and avoid errors.

These cleaning techniques improve the quality and robustness of the data for further analysis, demonstrating a commitment to high standards in data management. After cleaning, and given the large amount of emotional data provided by FaceReader, an average was calculated for each individual emotion across all samples. This allowed each student to be represented by the average of their samples for each of the 8 emotions, along with their sex, which was used for the neural network design.

Machine Learning

Machine Learning has emerged as an effective field driving significant advancements in a wide variety of applications. This computational approach relies on programming a computer with specific, easily transferable knowledge, without requiring a continuous learning process (Hildebrandt, 2017), and using these patterns to make decisions or predictions.

In this section, we will explore how ML is transforming our approach to various highly complex problems using extremely diverse and unstructured datasets (Chen and Cui, 2020), and how this technology can be applied in various areas. The study will focus on presenting a key model in this field: ANN, with special attention to the Multilayer Perceptron (MLP).

The fundamental structure of ANN, and how it is inspired by the functioning of the human brain to recognize patterns in data will be presented. Subsequently, the definition and operation of the MLP, a specific type of ANN known for its ability to model nonlinear relationships between input features and output results, will be delved into.

Throughout this section, we will be shown how the MLP is composed of layers of interconnected neurons, each playing a crucial role in the learning and prediction process. Furthermore, it will explore how the MLP uses techniques such as backpropagation to adjust the weights of neural connections and enhance its ability to make accurate predictions.

In summary, this section offers a detailed look at the structure and operation of ML, with a specific focus on the MLP model. Through this analysis, it is expected to provide a solid understanding of how ANNs, such as the MLP, are driving innovation in the field of ML and transforming the way we interact with data.

But what are ANN?

Artificial Neural Networks are a computational model inspired by the structure and functioning of the human brain, designed to recognize patterns. This type of network is composed of a series of interconnected nodes, also known as 'neurons.' These connections are key for the network to learn to recognize complex patterns through exposure to input data. (LeCun et al., 2015)

In the field of ML, one of the most commonly used types of ANN is the MLP, a form of feedforward neural network consisting of multiple layers of interconnected neurons. The MLP stands out for its ability to learn complex representations of input data and model nonlinear relationships between input features and output labels. (Jaramillo and Rüger, 2023)

Regarding its structure, an ANN generally consists of three types of layers. The first is the input layer, which receives raw data, similar to how human senses operate by receiving stimuli from the outside. The second consists of one or multiple hidden layers, where these data are processed through mathematical computations. In the case of the MLP, these hidden layers enable the model to learn more abstract and complex representations of the input data (Jaramillo and Rüger, 2023). Finally, the output layer provides the result of the network's processing, which may manifest as classification or prediction, among other forms. (Bishop, 2006)

Each neuron within these layers plays a crucial role: it receives multiple inputs, combines them using various methods, performs mathematical operations, and then transmits the result through an activation function. In the case of the MLP, activation functions introduce nonlinearities into the model, a fundamental aspect for learning and modeling complex patterns. Common examples of activation functions used in the MLP include logistic sigmoid, hyperbolic tangent, and ReLU (Rectified Linear Unit), among others. (Jaramillo and Rüger, 2023)

The learning process in ANN, including the MLP, is carried out by adjusting the weights of connections between neurons. This adjustment is often done through a method known as "backpropagation." In this process, the output error—the difference between the expected and obtained result—is used to modify the weights so that the error is reduced in future operations. The MLP, like other types of ANN, uses backpropagation to optimize weights during training and improve its ability to make accurate predictions. (Jaramillo and Rüger, 2023)

Thanks to their ability to learn from large volumes of data and recognize subtle patterns, ANN, including the MLP, are used in a wide range of applications in the field of ML, such as image recognition, natural language processing, time series, and many more. (Goodfellow et al., 2016).

In consideration of the above, and given the importance based on the Neuromathematical study, a predictive model is built to determine students' performance based on their sex and the emotional data provided by FaceReader.

Design and Optimization of a Neural Network for Binary Classification: Network Architecture Preprocessing, Data Balancing, Training, and Validation.

This study focuses on designing a deep ANN to perform binary classification tasks with a high degree of accuracy. The network is built using the TensorFlow and Keras frameworks in a Python environment. For information processing, the data obtained includes labels of emotions detected by FaceReader and the subject's sex. These data are extracted from an extensive database contained in an Excel spreadsheet, as shown in Table 1.

Model Features

Input Variable	Description
Angry	Emotion of anger
Contempt	Emotion of contempt
Disgusted	Emotion of disgust
Happy	Emotion of happiness
Neutral	Neutrality
Sad	Emotion of sadness
Scared	Emotion of fear
Surprised	Emotion of surprise
Sex	Sex of the individual (0: Man, 1: Woman)
Target Variable	**Description**
Performance	Task performance (1: Approved, 0: Unapproved)

Table 1. Model Features. Self-created (2024).

To ensure that the magnitudes of the features do not bias the model, they are normalized using the StandardScaler from Keras. Additionally, recognizing the importance of class balancing for predictive accuracy, ADASYN (Adaptive Synthetic Sampling Approach for Imbalanced Learning) is applied. This advanced oversampling technique adjusts the initial data classified as 'passed' and 'not passed,' increasing the emotional frequencies of those who did not pass. Consequently, based on the significant disproportion, it balances the data with synthetic examples to equalize the representation of the classes in the training data, as exemplified in Figure 5.

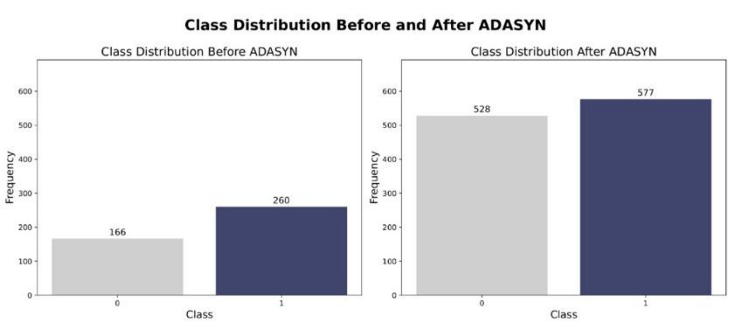


Figure 5. Class Distribution Before and After Adasyn.

Source: Self-Created (2024).

A reproducible environment is established by setting a random seed before training. The model is compiled with a binary crossentropy loss function, suitable for binary classification tasks, and an Adam optimizer (Adaptive Moment Estimation), chosen for its efficiency in weight adjustment.

The model is trained for 1000 epochs, allowing adequate convergence of the parameters. Each epoch represents a complete iteration over the training dataset, enabling the model to adjust its weights and biases to minimize the loss function. During training, the parameters are adjusted using a training dataset (80%) and the network is validated on a separate dataset (20%) to monitor the model's generalization. This split is achieved by using the train test split function, which divides the data into training and testing sets with a test size of 20%, ensuring a reliable evaluation of the model's performance. This process is conducted silently verbose=0 to focus on the quantitative evaluation of the model's performance, rather than real-time training metrics.

A batch size of 64 samples is used to balance computational efficiency and gradient stability during training. Using this relatively small batch size helps improve the model's generalization by preventing overfitting on the training data. The network architecture consists of a well-structured sequence of dense layers and batch normalization layers, designed to process the datasets effectively.

The input layer serves as the gateway to the model, receiving data with a total of nine features. Following this initial layer, batch normalization is incorporated into the first four layers, normalizing the inputs to a mean of zero and a variance of one, as detailed in Table 2. This stabilizes learning and improves the model's performance.

Term	Formula	Description
Batch Normalization	$\hat{z}^{(k)} = \frac{z^{(k)} - \mu_B}{\sqrt{\sigma_B^2 + \varepsilon}}$	Normalizes the inputs $z^{(k)}$ of each mini-batch B, where μ_B is the mini-batch mean and σ_B^2 is the variance. ε is a small constant to avoid division by zero.
ReLU Activation Function	$\mathrm{ReLU}(x) = \max(0, x)$	The ReLU (Rectified Linear Unit) function is an activation function that returns x if x is positive and 0 if x is negative.
L2 Regularization	$\frac{\lambda}{2}\|w\|^2$	Penalizes large values of the weights w in the network, where λ is the regularization parameter and $\|w\|^2$ is the sum of the squares of the weights.
Sigmoid Function	$\sigma(x) = \frac{1}{1+e^{-x}}$	The sigmoid function converts input values x into a range (0, 1), especially useful for binary classification models.

Table 2. Formula Table.

Source: Self-Created (2024).

A series of dense layers, detailed in Table 3, increase the model's capacity to extract complex features and underlying patterns in the data. Each of these dense layers is activated using the 'ReLU' function, which is mathematically defined as shown in Table 2. The 'ReLU' function is known for its efficiency and effectiveness in mitigating issues such as the vanishing gradient problem in deep neural networks.

Model Summary

	Layer (type)	Output Shape	Trainable Params	Activation Function
Input	InputLayer	(None, 9)	0	N/A
BatchNorm 1	BatchNormalization	(None, 9)	36	N/A
Dense 1	Dense	(None, 256)	2560	ReLU
BatchNorm 2	BatchNormalization	(None, 256)	1024	N/A
Dense 2	Dense	(None, 128)	32896	ReLU
BatchNorm 3	BatchNormalization	(None, 128)	512	N/A
Dense 3	Dense	(None, 64)	8256	ReLU
BatchNorm 4	BatchNormalization	(None, 64)	256	N/A
Dense 4	Dense	(None, 32)	2080	ReLU
BatchNorm 5	BatchNormalization	(None, 32)	128	N/A
Output	Dense	(None, 1)	33	Sigmoid

Table 3. Model Summary: Neural Network Layers.

Source: Self-created (2024).

In addition to Batch Normalization, L2 regularization (Ridge Regularization) is applied to each dense layer to mitigate overfitting by penalizing excessively large network weights. This technique prevents excessive reliance on particular inputs and thus improves generalization to new data (see the formula in Table 2).

The network culminates in an output layer with a single neuron that uses the 'Sigmoid' activation function, whose formula is shown in Table 2. This setup is mainly used in models where probabilities are required, as its output ranges between 0 and 1. It is typical of binary classification tasks, where the output represents the probability of belonging to one of two possible categories.

Figure 6 details the network architecture, showing the input layer, the four dense layers, and the output layer.

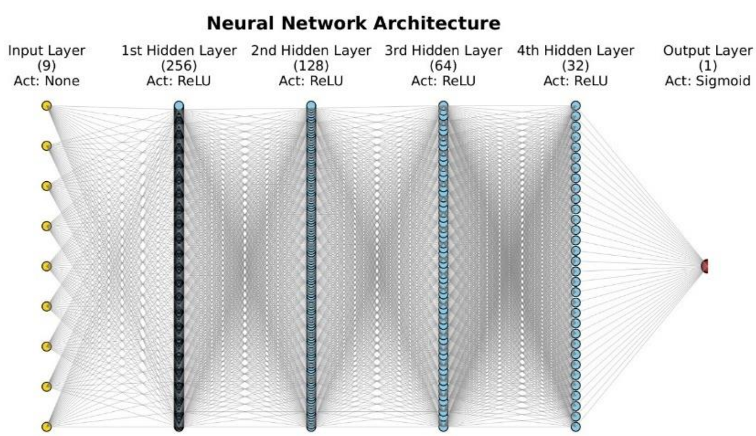


Figure 6. Network Architecture. Source: Self-created (2024).

Once the network is trained, it is evaluated on a test set. Accuracy and loss are recorded as key performance metrics. For a more detailed evaluation, the confusion matrix is employed, providing a deeper insight into the model's performance by considering precision, recall, and F1 score for both classes.

The confusion matrix shown in Figure 7 illustrates how the model classifies instances between the two possible categories. In this matrix:

- **100 true negatives (TN):** Instances correctly identified as class 0.
- **102 true positives (TP):** Instances correctly identified as class 1.
- **13 false negatives (FN):** Instances of class 1 incorrectly classified as class 0.
- **6 false positives (FP):** Instances of class 0 incorrectly classified as class 1.

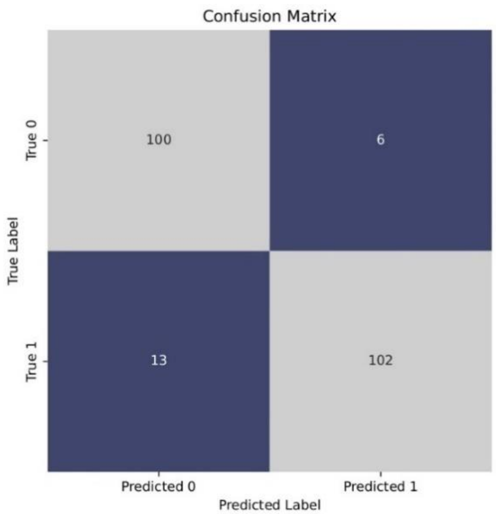


Figure 7. Confusion Matrix.

Source: Self-created (2024).

The detailed classification report, shown in Table 4, extends this information by calculating and clearly displaying these metrics for each class. It helps to assess the balance between correctly identifying positive instances and avoiding incorrectly classifying negatives as positives.

Classification Report

	Precision	Recall	F1-score	Support
0	0.88	0.94	0.91	106
1	0.94	0.89	0.91	115
Macro Avg	0.91	0.92	0.91	221
Weighted Avg	0.92	0.91	0.91	221

Accuracy: 0.91

Table 4. Classification Report.

Source: Self-created (2024).

This report reflects the following metrics for each class:

- **Precision:** The model's ability to identify only the relevant examples. For class 0, it is 0.88, indicating that 88% of the instances classified as class 0 are correct. For class 1, it is 0.94, with 94% precision in correctly identifying class 1.

- **Recall (Sensitivity):** The model's ability to find all relevant examples. For class 0, the model has a recall of 0.94, meaning it captures 94% of all actual instances of class 0. For class 1, the recall is 0.89.

- **F1-Score:** A measure that combines precision and recall into a single metric that weighs both equally. The F1-score for both classes is 0.91, suggesting a good balance between precision and recall.

- **Support:** reflects the number of actual cases in the test set, where there are 106 instances of class 0 and 115 of class 1. Although oversampling was used to balance the classes in the training set (577 for class 1 and 528 for class 0), the reported metrics correspond only to the test set, which is smaller and not necessarily balanced.

The overall **Accuracy metric** of the model is 91%, indicating high efficiency in correctly classifying instances. The macro and weighted averages reflect consistent and balanced performance across the classes, demonstrating that the model is effective in handling imbalanced classes.

This detailed analysis provides a clear understanding of how the model handles different types of classifications, highlighting areas of strength and opportunities for future improvements.

Additionally, to gain a more comprehensive understanding of the model's behavior at different classification thresholds, the ROC curve, exemplified in Figure 8, is used. It is particularly useful for visualizing the model's ability to separate the classes.

The ROC Curve illustrates the relationship between the true positive rate (TPR) and the false positive rate (FPR) at different decision thresholds. In this case, the area under the curve (AUC) is 0.96, indicating optimal model performance. An AUC of 1.0 represents a perfect classifier, while an AUC of 0.5 would indicate performance no better than a coin toss. The orange curve rises sharply towards the top of the graph, reflecting a high true positive rate (sensitivity) with a low false positive rate.

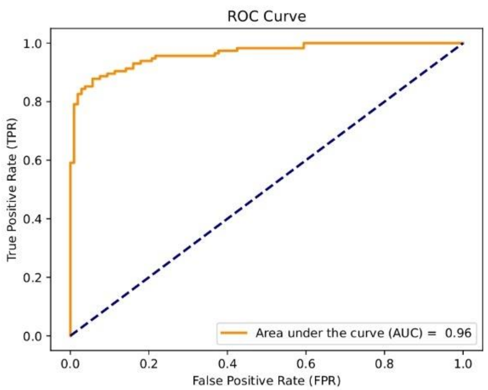


Figure 8. ROC Curve.

Source: Self-created (2024).

This high **AUC** highlights the model's ability to correctly classify positive and negative instances, demonstrating that the model is robust and reliable under various operational conditions. Including this metric in the model evaluation provides a clear measure of its ability to handle the trade-off between capturing true positives and avoiding false positives, which is crucial for applications where detection precision is vital.

During model training, accuracy and loss were monitored at each epoch for both the training and validation sets. Figure 9 exemplifies this, providing a clear view of how the model learns and adjusts over time.

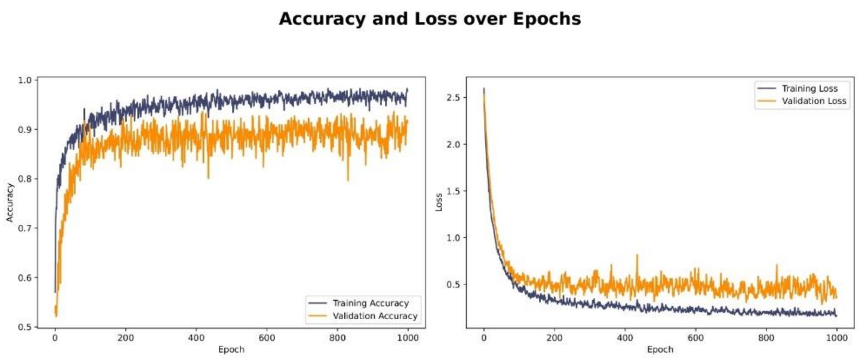


Figure 9. Accuracy and Loss Over Epochs.

Source: Self-created (2024).

As detailed in the graph on the left, the training and validation accuracy increase notably in the initial epochs and stabilize near 1.0 for training and 0.9 for validation, indicating a high level of learning and good model generalization. On the other hand, the graph on the right shows how the loss decreases rapidly in the initial epochs and stabilizes towards the end, with the training loss consistently lower than the validation loss, suggesting that the model is not significantly overfitting.

In conclusion, these graphs provide clear evidence of the model's robustness and reliability, demonstrating its ability to learn effectively and make accurate and consistent predictions.

Discussion

In this study, a predictive model was developed using emotional data obtained through FaceReader and demographic information to predict student performance. The key input variables included a range of emotions ('Angry', 'Contempt', 'Disgusted', 'Happy', 'Neutral', 'Sad', 'Scared', 'Surprised') and the sex of the students, while the target variable was task performance, classified as either approved or unapproved.

1. Data Imbalance and ADASYN

The original dataset showed a significant imbalance, with fewer cases of failed tasks (166) compared to passed tasks (260). To address this imbalance, the Adaptive Synthetic Sampling (ADASYN) technique was applied, generating a more balanced dataset with 528 cases of failed tasks and 577 of passed tasks. This balance was crucial for training a robust model that did not favor the majority class, thereby improving generalization to new data.

2. Model Architecture

The model consists of multiple dense layers interspersed with batch normalization layers. The dense layers utilize ReLU activation functions, which are effective for handling non-linearity, while the final layer employs a sigmoid activation function suitable for binary classification tasks. The inclusion of batch normalization layers stabilizes and accelerates the training process by normalizing the inputs of each mini-batch.

3. Performance Metrics

The model achieved an accuracy of 91%, with precision and recall values indicating strong performance in both classes. The classification report showed a precision of 0.88 and 0.94 for the fail and pass classes, respectively, with recall values of 0.94 and 0.89. The F1-scores were balanced at 0.91 for both classes, underscoring the model's reliability. Additionally, the ROC curve, with an AUC of 0.96, further confirmed the model’s high discriminative power.

4. Insights from the Confusion Matrix

The confusion matrix revealed that the model correctly classified 100 cases of failed tasks and 102 cases of passed tasks. Misclassifications were relatively few, with 6 false positives and 13 false negatives. The slightly higher rate of false negatives suggests that further adjustments may be needed to reduce undetected positive cases.

5. Training and Validation

The training process demonstrated convergence, as indicated by the precision and loss curves. The training and validation precision curves showed a steady increase, while the loss curves consistently decreased, suggesting that the model effectively learned underlying patterns without significant overfitting.

According to Table 5, the Correlation Matrix allows us to identify the emotions that have both positive and negative impacts on student performance. The main findings are presented below:

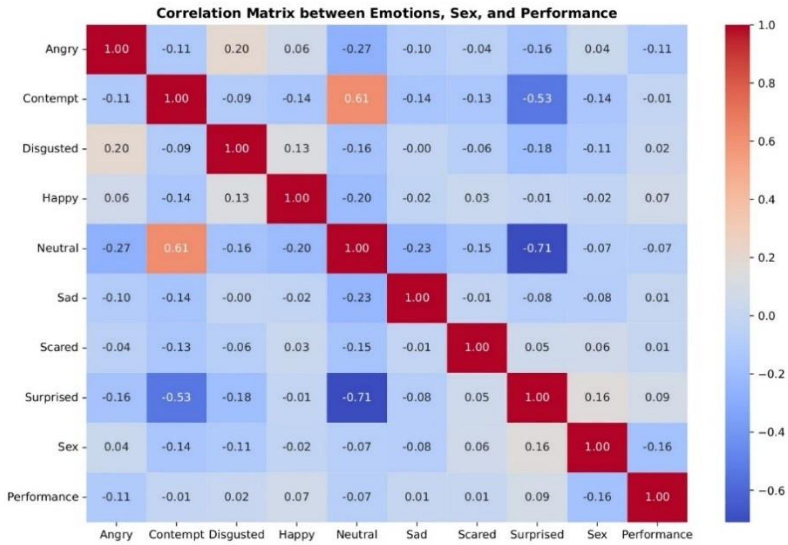


Table 5. Correlation Matrix Between Emotions, Sex, And Performance.

Source: Self- Created (2024).

Indicators of Low Performance (Unapproved): Students underperformed due to the negative influence of emotions such as anger and neutrality. The most notable negative correlation was with the emotion of anger (-0.11), which can act as a distracting or demotivating factor, affecting students' ability to perform well on tasks. Neutral expression also had a moderate negative correlation (-0.07), suggesting these students may be less engaged or motivated, leading to poor performance.

Indicators of High Performance (Approved): Positive emotions, such as happiness, were associated with better performance, although the correlation was small (0.07). Happiness may be linked to increased motivation, concentration, and overall well-being, which are important factors for good academic performance. Surprise also showed a small positive correlation (0.09), possibly indicating that genuine interest or curiosity during the task can positively impact performance.

Emotions play a significant role in student performance. Emotions such as happiness and surprise are associated with better performance, while anger and neutrality have an adverse impact. Additionally, the sex of the student appears to influence performance, suggesting the need for specific interventions to address these differences. Fostering an environment that promotes positive emotions and effectively manages negative ones could help improve students' academic results.

The developed predictive model proved effective in predicting student performance based on emotional and demographic data. The use of data balancing techniques such as ADASYN was crucial for ensuring a balanced and robust model. The model's architecture, with its multiple dense and normalization layers, allowed for effective and stable learning. The performance metrics and confusion matrix underscore the model's accuracy and reliability, although the need for additional adjustments to reduce the false negative rate was identified. Overall, the study demonstrates the potential of emotional and demographic data in predicting academic performance, offering a promising approach for future research and practical applications.

Analysis of Feature Importance

As shown in Figure 10, the analysis of the importance of the characteristics using Permutation Importance and the coefficients of logistic regression provides a comprehensive view of how different emotions and sex influence the prediction of performance.

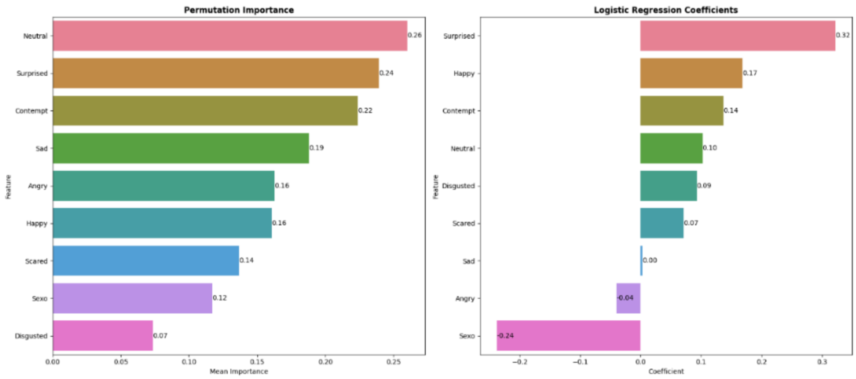


Figure 10. Feature Importance using Permutation Importance and Logistic Regression Coefficients.

Source: Self-created (2024).

Permutation Importance reveals that the "Neutral" emotion is the most important feature, with a mean importance of 0.26, indicating that permuting its values significantly impacts the model’s performance. This is followed by the emotions "Surprised" (0.24) and "Contempt" (0.22), also standing out as key factors in performance prediction. Emotions such as "Sad" (0.19), "Scared" (0.14), "Happy" (0.16), and "Angry" (0.16) also show considerable importance, though to a lesser degree. In contrast, the feature "Sex" (0.12) and the emotion "Disgusted" (0.07) have a relatively smaller impact according to this method.

Permutation Importance is a valuable technique for assessing feature importance in ML models, as it measures the impact on model performance when the values of a feature are permuted. This technique has been widely discussed and applied in various studies, as described in the work of Fisher et al., (2019) on interpreting predictive models and measuring feature importance.

On the other hand, the coefficients of logistic regression provide information on the direction and magnitude of each feature’s effect on the probability of approval. The emotions "Surprised" (0.32), "Happy" (0.17), and "Contempt" (0.14) have significant positive coefficients, indicating that an increase in these values is associated with a higher probability of approval. The emotions "Scared" (0.07) and "Disgusted" (0.09) also have positive effects, though more moderate. However, the negative coefficient for "Sex" (-0.24) suggests a considerable inverse relationship, indicating that sex is a significant factor in decreasing the probability of approval. Emotions such as "Angry" (-0.04) also show a negative impact, but of lesser magnitude.

Logistic regression is a widely used technique for modeling the relationship between a binary dependent variable and one or more independent variables. This methodology has been fundamental in numerous studies and applications, as detailed in the article by Kleinbaum and Klein (2010) on applied logistic regression.

The combination of Permutation Importance and logistic regression offers a comprehensive perspective on the most influential features in predicting student performance. The emotions "Neutral", "Contempt", and "Surprised" emerge as key factors, while sex also plays an important role, though with a negative influence.

Neuromathematical Approach

The Neuromathematical approach was essential to the success of the predictive model developed in this study. By integrating emotional data captured in real-time through FaceReader and combining them with demographic variables such as sex, the model allowed for a more accurate prediction of academic performance. The Neuromathematical analysis facilitated the identification of emotional and cognitive patterns that might have gone unnoticed in a traditional analysis. Emotions such as surprise and happiness were positively correlated with better performance, while negative emotions, such as anger, showed a negative impact on students' ability to concentrate and solve mathematical problems.

The model's ability to capture the interaction between emotions and sex is a significant advancement. The analysis showed that negative emotions, such as anger, reduced the cognitive activity required for problem-solving, highlighting the relevance of the Neuromathematical approach in predicting performance. By integrating this neuro-emotional dimension into the model, a more robust and personalized tool has been created to predict academic success in mathematics.

Limitations and Future Research

Despite the promising results, the study presents certain limitations regarding the generalization of findings. The task focused on constructing a geometric cone, which may not fully represent the range of mathematical skills that students develop in other areas of the curriculum. Nevertheless, this Neuromathematical approach provides a solid foundation for future research aiming to apply this model to other mathematical contexts, such as algebra and calculus, where students face different emotional and cognitive challenges (Ashcraft & Moore, 2009).

In addition, emotional analysis using facial recognition technologies could allow teachers to personalize pedagogical interventions based on the students' emotional states in real-time. For example, in adaptive learning environments, where students receive tasks tailored to their competence levels, activities could be adjusted not only based on cognitive progress but also on the emotions displayed when facing academic challenges. This strategy offers a more holistic approach to enhancing both academic performance and student well-being.

Practical Implications

This study offers important practical implications for the educational field. By integrating real-time captured emotions and demographic data such as sex, educators can adjust their teaching strategies to accommodate the emotional and cognitive needs of each student. This can have a direct impact on early intervention, as the developed model allows for the identification of students at risk of low academic performance before their difficulties worsen.

Furthermore, the model provides a powerful tool for implementing personalized support, adjusting academic activities not only based on cognitive progress but also on the emotions that students experience while performing tasks. This approach promotes a more inclusive learning environment, sensitive to emotional differences that may vary according to sex.

Future Directions for Research

The predictive model developed, which integrates emotions, sex, and academic performance, has the potential to be expanded by incorporating other emotional or biometric indicators, such as heart rate or electrodermal activity. Future studies could also explore the real-time application of these models in classroom environments, allowing teachers to adjust their instruction based on students' emotional states.

Longitudinal studies that follow students over several years would provide valuable insights into the evolution of emotions and their impact on learning at different stages of academic development. This would help validate the applicability of predictive models across a wider range of educational contexts, further enhancing their ability to personalize learning based on sex and academic performance.

Conclusion

This study demonstrates the value of the Neuromathematical approach in understanding mathematical learning. By integrating neuroscience and mathematics didactics through an innovative Theoretical Framework, a more comprehensive view is provided of how students face mathematical challenges. The findings suggest that emotional regulation is a crucial component of academic success, especially in tasks requiring high levels of concentration and problem-solving.

The Neuromathematical approach not only enriched the interpretation of the results but also significantly improved the accuracy of the predictive model. By incorporating emotional and cognitive variables grounded in neuroscience, this study presents an innovative approach that can be extended to other learning domains, enhancing the personalization and effectiveness of pedagogical interventions.

Although this study focused on a specific geometric task, its findings can be generalized to a wide range of mathematical and academic contexts if carefully addressed in future studies. Replicating this approach across different areas of mathematics and other academic domains would broaden our understanding of how emotions influence academic performance, offering useful predictive tools to personalize teaching and improve student success in diverse educational settings.

This study highlights a significant relationship between emotions, sex, and academic performance in the geometric task. Emotions such as joy and surprise were correlated with greater success, while negative emotions such as fear and anger were linked to unapproval of the task, reinforcing existing literature on the inhibitory impact of these emotions on mathematical cognition (Young et al., 2012). Additionally, the analysis of specific traits, such as neutrality and contempt, emphasizes their crucial role as predictors. Finally, the high accuracy of the model underscores the potential of emotional and demographic data to predict academic success and personalize educational interventions to improve student outcomes.

Clarification note on the use of the term "Sex". In this research, the term "Sex" is used to refer to the biological differences between males and females and should not be confused with the concept of "Gender," which refers to social and cultural constructions around identity and roles. The analysis based on Sex focuses on identifying biological and emotional patterns that may influence academic performance and problem-solving in mathematics. The inclusion of Sex as an explanatory variable responds to scientific evidence of significant differences in the management of emotions and cognitive processes between males and females, contributing to a deeper understanding of how these differences impact learning. However, we acknowledge the complexity of the subject and the importance of addressing these aspects with sensitivity and precision in future research.

References

Ashcraft, M. H., & Moore, A. M. (2009). Mathematics anxiety and the affective drop in performance. Journal of Psychoeducational Assessment, 27(3), 197-205. https://doi.org/10.1177/0734282908330580

Battista, M. T. (2007). The development of geometric and spatial thinking. Second handbook of research on mathematics teaching and learning, 843-908.

Bishop, C. M. (2006). Pattern Recognition and Machine Learning. Springer.

Bosch, N., D'Mello, S., Baker, R. S., Ocumpaugh, J., Shute, V., Ventura, M., ... & Zhao, W. (2016). Detecting student emotions in computer-enabled classrooms. Proceedings of the 2016 CHI Conference on Human Factors in Computing Systems, 4001-4006. https://doi.org/10.1145/2858036.2858537

Chen, F., & Cui, Y. (2020). Utilizing Student Time Series Behaviour in Learning Management Systems for Early Prediction of Course Performance. Journal of Learning Analytics, 7(2), 1-17. https://doi.org/10.18608/jla.2020.72.1

Clements, D. H., & Sarama, J. (2011). Early childhood mathematics education research: Learning trajectories for young children. Routledge.

Cruz, E., González, M., & Rangel, J. (2022). Técnicas de Machine Learning aplicadas a la evaluación del rendimiento y a la predicción de la deserción de estudiantes universitarios, una revisión. Prisma Tecnológico, 13(1), 77-87. https://doi.org/10.33412/pri.v13.1.3039

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2004). Three parietal circuits for number processing. Cognitive Neuropsychology, 21(5), 487-506. https://doi.org/10.1080/02643290244000239

D'Mello, S., & Graesser, A. (2012). Dynamics of affective states during complex learning. Learning and Instruction, 22(2), 145-157. https://doi.org/10.1016/j.learninstruc.2011.10.001

D'Mello, S., Dieterle, E., & Duckworth, A. (2017). Advanced, analytic, automated (AAA) measurement of engagement during learning. Educational Psychologist, 52(2), 104-123. https://doi.org/10.1080/00461520.2017.1281747

Ekman, P. (2003). El rostro de las emociones, volumen 7. RBA Libros S.A, Barcelona.

Ekman, P. (2003). Emotions revealed: Recognizing faces and feelings to improve communication and emotional life. Henry Holt and Company.

Ekman, P. y Davidson, R. J. (1994). The nature of emotion: Fundamental questions (pp. 20-24). Nueva York: Oxford University Press.

Else-Quest, N. M., Hyde, J. S., & Linn, M. C. (2010). Cross-national patterns of gender differences in mathematics: A meta-analysis. Psychological Bulletin, 136(1), 103-127. https://doi.org/10.1037/a0018053

Fisher, A., Rudin, C., & Dominici, F. (2019). All Models are Wrong, but Many are Useful: Learning a Variable's Importance by Studying an Entire Class of Prediction Models Simultaneously. Journal of machine learning research: JMLR, 20, 177.

Ganley, C. M., & Lubienski, S. T. (2016). Mathematics confidence, interest, and performance: Examining gender patterns and reciprocal relations. Learning and Individual Differences, 47, 182-193. https://doi.org/10.1016/j.lindif.2016.01.002

Giraldo-Rojas, J. D. Zabala-Jaramillo, L. A. Parraguez, M. C. (2017). Neuromatemática un estudio interdisciplinario: el caso de las emociones expresadas en la construcción del paralelepípedo.

Neuromathematics an interdisciplinary study: the case of the emotions expressed in the construction of the parallelepiped. Scientia et Technica. 26(03), 378-390. https://doi.org/10.22517/23447214.2475
Goetz, T., Bieg, M., Lüdtke, O., Pekrun, R., & Hall, N. C. (2013). Do girls really experience more anxiety in mathematics? Psychological Science, 24(10), 2079-2087. https://doi.org/10.1177/0956797613486989
Gómez, J. Marcos-Merino, J. Méndez, F. Mellado, V. Esteban, M. (2019). Emociones académicas y aprendizaje de biología, una asociación duradera. Enseñanza de las Ciencias. 37(2), 43-61. https://doi.org/10.5565/rev/ensciencias.2598
Goodfellow, I., Bengio, Y., & Courville, A. (2016). Deep Learning. MIT Press.
Harley, J. M., Bouchet, F., Hussain, M. S., Azevedo, R., & Calvo, R. A. (2015). A multi-componential analysis of emotions during complex learning with an intelligent multi-agent system. Computers in Human Behavior, 48, 615-625. https://doi.org/10.1016/j.chb.2015.02.013
Hernández Sampieri, R. Fernández Collado, C. Baptista Lucio, P. (2014). Metodología de la investigación. Interamericana Editores S.A.
Herodotou, C., Naydenova, G., Boroowa, A., Gilmour, A., & Rienties, B. (2020). How Can Predictive Learning Analytics and Motivational Interventions Increase Student Retention and Enhance Administrative Support in Distance Education?. Journal of Learning Analytics, 7(2), 72-83. https://doi.org/10.18608/jla.2020.72.4
Hildebrandt, M. (2017). Learning as a Machine: Crossovers Between Humans and Machines. Journal of Learning Analytics, 4(1), 6–23. https://doi.org/10.18608/jla.2017.41.3
Immordino-Yang, M. H., y Damasio, A. (2007). We Feel, Therefore We Learn: The Relevance of Affective and Social Neuroscience to Education. Mind, Brain, and Education, 1(1), 3–10. https://doi.org/10.1111/j.1751-228x.2007.00004.x
Instituto Colombiano para la Evaluación de la Educación (Icfes). (2020). Informe Nacional de Resultados para Colombia - PISA 2018.
Jaramillo, H. Rüger. A. (2023). Machine Learning for Science and Engineering. 1 edition. Society of Exploration Geophysicists (SEG), Houston, USA.
Kleinbaum, D. G. Klein. M. (2010) Logistic Progression: A Selft Learning Text. Springer Science & Business Media. https://doi.org/10.1007/978-1-4419-1742-3
Laborde, C., Kynigos, C., Hollebrands, K., & Strässer, R. (2006). Teaching and learning geometry with technology. Handbook of research on the psychology of mathematics education, 275-304.
LeCun, Y., Bengio, Y. & Hinton, G. Deep learning. Nature. 521, 436–444 (2015). https://doi.org/10.1038/nature14539
Marcos-Merino, J. M., Esteban Gallego, R. y Ochoa de Alda, J. A. G. (2022). Conocimiento previo, emociones y aprendizaje en una actividad experimental de ciencias. Enseñanza de las Ciencias, 40(1),107-124. https://doi.org/10.5565/rev/ensciencias.3361
Noldus. (2016). FaceReader resources. https://www.noldus.com/facereader/resources
Núñez-Peña, M. I., Suárez-Pellicioni, M., & Bono, R. (2013). Effects of math anxiety on student success in higher education. International Journal of Educational Research, 58, 36-43. https://doi.org/10.1016/j.ijer.2012.12.004
Pekrun, R. (2006). The control-value theory of achievement emotions: Assumptions, corollaries, and implications for educational research and practice. Educational Psychology Review, 18(4), 315-341. https://doi.org/10.1007/s10648-006-9029-9
Reeve, J. (2010). Motivación y emoción. McGraw-Hill/Interamericana Editores S.A. de C.V.
Rouhiainen L. P. (2018). Inteligencia artificial, 101 cosas que debes saber hoy sobre nuestro futuro. Planeta Libros, España. ISBN: 978-84-17568-08-5

Skiendziel, T., Rösch, A. G., & Schultheiss, O. C. (2019). Assessing the convergent validity between the automated emotion recognition software Noldus FaceReader 7 and Facial Action Coding System Scoring. PLoS ONE. 14(10). https://doi-org.ezproxy.eafit.edu.co/10.1371/journal.pone.0223905

Skiendziel, T., Rösch, A. G., & Schultheiss, O. C. (2019). Assessing the convergent validity between the automated emotion recognition software Noldus FaceReader 7 and Facial Action Coding System Scoring. PLoS ONE, 14(10), e0223905. https://doi.org/10.1371/journal.pone.0223905

Yl, M. & Kuilenburg, H.. (2005). The FaceReader: Online Facial Expression Recognition. Proceedings of Measuring Behavior. Wageningen.

Young, C. B., Wu, S. S., & Menon, V. (2012). The neurodevelopmental basis of math anxiety. Psychological Science, 23(5), 492-501. https://doi.org/10.1177/0956797611429134

Zabala-Jaramillo, L., Díaz Barriga, E., Roa-Fuentes, S., Parraguez, M., Laborde, C., Laborde, J. M., Fayó, A., Morales, A., Huincahue, J., y Rodríguez, R. (2017). Un nuevo marco conceptual: Modelación y Representación con Geometría Dinámica y Matemática Condicional. En N. Hincapié (presidencia). I Congreso Internacional de Cabri Universidad de Medellín. Congreso llevado a cabo en Medellín, Colombia.

Zhou, G., Cheng, G., & Zhang, J. (2019). The multimodal emotion recognition network: Recognizing human emotions with multimodal signal fusion in intelligent tutoring systems. IEEE Transactions on Learning Technologies, 12(4), 469-482. https://doi.org/10.1109/TLT.2019.2902543

Zhu, A., Boonipat, T., Cherukuri, S., & Bite, U. (2023). Defining Standard Values for FaceReader Facial Expression Software Output. Aesthetic Plastic Surgery. https://doi-org.ezproxy.eafit.edu.co/10.1007/s00266-023-03468-y.