Journal of Posthumanism
2025
Volume: 5, No: 1, pp. 720–744
ISSN: 2634-3576 (Print) | ISSN 2634-3584 (Online)
posthumanism.co.uk

DOI: https://doi.org/10.63332/joph.v5i1.603

Fuzzy Time Series Modeling for Predicting Economic Trends: A Mathematical Exploration

Yogeesh N[1], Suleiman Ibrahim Shelash Mohammad[2,3], Khaleel Ibrahim Al- Daoud[4], Dr N Raja[5], Hanumantha Reddy D T[6], P. William[7], Asokan Vasudevan[8], Muhammad Turki Alshurideh[9]

Abstract

In fact, this research investigates the potential use of FTS models for economic indicators prediction in the Asian area, particularly the GDP growth. However, traditional forecasting approaches tend to fail to represent the uncertainty and nonlinearity of the economic data, especially in crisis periods. By applying fuzzy inference rules and defuzzification to create crisp forecasts from fuzzy sets, FTS models capitalize on the principles of fuzzy logic to provide a versatile and adjustable framework for predicting economic outcomes. Using FTS for GDP growth forecasting, the study utilizes a dataset that includes GDP growth and inflation rates over the years 2010 to 2022. FTS models adapt well to uncertain and volatile economic variables, and although challenges in determining the most suitable fuzzy membership function and fuzzy rules exist, the results demonstrate their potential to extract process understanding. The model gave an accuracy (Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE)) of about 5.52 and 5.81, which shows okay predictions, but also we can work on this, and improve our model. To demonstrate the performance of the FTS, we compare to traditional econometric models and machine learning techniques, and highlight the strengths and weaknesses of FTS. This study finally concludes that there is significant potential for evolving FTS models for economic forecasting while future research avenues can assist in developing hybrid fuzzy models, fuzzy clustering and real-time forecasting using FTS models.

Keywords: *Fuzzy Time Series (FTS), Fuzzy Inference System (FIS), Economic Forecasting, GDP Growth Prediction, Fuzzy Logic, Defuzzification, Performance Evaluation, Machine Learning in Economics.*

Introduction

Overview of Economic Prediction Models

Building economic trend forecasting has used disparate statistical models as the classical

[1] Department of Mathematics, Government First Grade College, Tumkur, Karnataka, India, Email: yogeesh.r@gmail.com
[2] Electronic Marketing and Social Media, Economic and Administrative Sciences Zarqa University, Jordan.
[3] Research follower, INTI International University, 71800 Negeri Sembilan, Malaysia, Email: dr_sliman@yahoo.com
[4] Department of Accounting– business school Faculties–Al Ahilya Amman University –Amman-Jordan, Email: K.aldaoud@ammanu.edu.jo
[5] Assistant Professor, Sathyabama Institute of Science and Technology, (Deemed to Be University), Department of Visual Communication, Chennai, Tamil Nadu, Email: rajadigimedia2@gmail.com
[6] Associate Professor of Mathematics, Govt. College for Women, Chintamani, Chikkaballapur, India, Email: hanumanthareddydt@gmail.com
[7] Department of Information Technology, Sanjivani College of Engineering, Savitribai Phule Pune University, Pune, India
[8] Faculty of Business and Communications, INTI International University, 71800 Negeri Sembilan, Malaysia, Email: asokan.vasudevan@newinti.edu.my
[9] Department of Marketing, School of Business, The University of Jordan, Amman 11942, Jordan, Email: m.alshurideh@ju.edu.jo

approach to extract future economical patterns from past observations. ARIMA (Autoregressive Integrated Moving Average) (Box, Jenkins, & Reinsel, 2008) is one of the most popular models among several models of time series analysis. The ARIMA Models prove useful for creating prediction models on stationary time series data which uses the past values (x) and has the lag of all past differences (p wherever required) to predict future outputs. The autoregressive term (p) of ARIMA model usually describes the relationship between an observation and a number of lagged observations. Another commonly used model in time series is the Exponential Smoothing models (Holt & Winters, 2007), it use weighted average of history points when the data has trend and seasonality.

These models have big limitations:

- **ARIMA models** require the data to be stationary, which is often not the case in economic data, making their application limited (Hyndman & Athanasopoulos, 2018).
- **Exponential smoothing models** struggle when dealing with highly volatile or irregular data (Makridakis et al., 2020).
- Both models are often inadequate in capturing the **non-linearity** and **uncertainty** inherent in economic systems (Tsaur& Tzeng, 2002).

Role of Fuzzy Logic in Economic Predictions

Fuzzy logic (Zadeh, 1965) is one of the most widely used approaches for dealing with the imprecision and vagueness that is often inherent in real-world data. Pragmatically, uncertainty is inherent in economic systems (for example, regarding fluctuating market conditions, political and social factors) that render conventional models ineffectual. Fuzzy logic is a kind of logic based on fuzzy set theory, which is able to represent imprecision in values in the form of linguistic terms, modelling linguistic uncertainty.

1. The Fuzzy Time Series (FTS) modelling is greatly assisted by fuzzy logic as it provides a socioeconomic approach to the prediction of timed-set data that accounts for the uncertainty seen in the behaviour of economic time series data (Zadeh, 1965).
2. The FTS methodology addresses non-linearity and dynamism which the ordinary linear models cannot (Kuo, Hwang, & Yang, 2006).
3. The Fuzzification of economic data allows for the modelling of vague trends and patterns, which makes FTS especially relevant for economic predictions (Chen, 2013).

Mathematical modelling in FTS includes:

1. **Fuzzification**: These numbers are crisp economic values, and membership functions are used to convert them into fuzzy numbers.

$$\mu_A(x) = \frac{1}{1+(x-c)^2}$$

   where $\mu_A(x)$ is the membership function for a fuzzy set A, here x is the crisp value, and c is the center of the fuzzy set.

2. **Defuzzification**: From fuzzy outputs back into crisp values One common approach is the centroid method:

$$\text{Crisp }(x) = \frac{\sum \quad x \cdot \mu(x)}{\sum \quad \mu(x)}$$

where $\mu(x)$ represents the membership function of fuzzy values x.

Objectives of the Study

The objective of this study is twofold:

1. To explore how Fuzzy Time Series (FTS) models can improve the accuracy of predicting economic trends, particularly for complex, non-linear, and uncertain economic data.
2. To analyse the mathematics structures and methods applied in FTS modelling such as fuzzification, fuzzy-rule based system, and chargers in combining fuzzy logic and machine learning to make robust economic prediction results.

Literature Review

Traditional Time Series Models in Economics

Econometric models, such as ARIMA and Vector Autoregressive Models (VAR), are frequently employed in traditional time series analysis, especially in the field of economics. The ARIMA model (Box, Jenkins, & Reinsel, 2008) operates under the assumption that the time series itself is linear and can be represented in the following form:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \epsilon_t$$

where y_t is the series at time $t, \phi_1, \phi_2, \ldots, \phi_p$ are the parameters, and ϵ_t is the error term.

In fact, ARIMA does have some drawbacks that can compromise its performance on non-stationary as well as volatile data. Vector Autoregressive (VAR) models are pioneered to characterize the interdependencies between multiple time series(Sims, 1980), represented as:

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + \epsilon_t$$

where Y_t is a vector of multiple time series variables at time t, and $A_1, A_2, \ldots, A_p$ are the matrices of coefficients.

Fuzzy Time Series (FTS) and its Application in Economics

Fuzzy Time Series (FTS) modelling has become popular, since it is a useful means to deal with the uncertainty of economic data. FTS is an extension of traditional time series models by using fuzzy sets to represent imprecise or vague information.

In the FTS framework, the time series is fuzzified where crisp values are mapped to fuzzy sets, followed by FIS prediction. In the case of inflation prediction, for example, FTS could create underlying domains that treat values as fuzzy classes by partitioning historical data into "Low," "Moderate," and "High."

A basic FTS model was proposed by Chen (1996) in the form of fuzzifying historical time series data and using a set of fuzzy rules to forecast future values. As such, the set of fuzzy rules can be expressed as:

$$IF\ Previous\ Value\ is\ Low\ THEN\ Next\ Value\ is\ Low$$

These fuzzy rules are processed using a fuzzy inference system.

Advancements in Fuzzy Logic for Time Series

Over the years fuzzy logic approaches have matured and incorporated with other advanced techniques such as machine learning and deep learning (Gama, 2019) to achieve higher accuracy in time series prediction. These advancements include:

Fuzzy Clustering: A technique used to cluster time series data according to fuzzy similarities (Bezdek, 1981)

Hybrids Fuzzy Models: Combines fuzzy logic with those of Artificial Neural Networks (ANNs) or Support Vector Machines (SVMs) (Mabrouk &Zghali, 2020).

The mathematical developments of fuzzy clustering are:

1. **Fuzzy C-Means (FCM)** algorithm for clustering economic time series data, which minimizes the objective function:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \epsilon_t$$

where y_t is the series at time t, $\phi_1, \phi_2, \dots, \phi_p$ are the parameters, and ϵ_t is the error term.

But ARIMA does not work well for non-stationary and volatile data. We can use VAR models to exploit multi-time series interdependencies (Sims, 1980), as expressed in:

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + \epsilon_t$$

where Y_t is a vector of multiple time series variables at time t, andhere, $A_1, A_2, \dots, A_p$ are the matrices of coefficients.

Theoretical Background

Fuzzy Logic Fundamentals

Fuzzy Sets and Fuzzy Numbers: This same definition can also extend to fuzzy logic. Membership functions assign a value for every member (also called an element) of the set to indicate the degree of membership of that element (Zadeh, 1965). In classical sets, an object either is or is not part of a set; in a fuzzy set, this membership has a value that can range in the interval [0,1], indicating partial belonging.

Mathematically, a fuzzy set A is represented as:

$$A = \{(x, \mu_A(x)) \mid x \in X\}$$

where X is the universe of discourse, and $\mu_A(x)$ is the membership function of element x in the fuzzy set A, with $\mu_A(x) \in [0,1]$ (Zadeh, 1965).

Fuzzy Numbers: A fuzzy number is a special type of fuzzy set used to represent numerical values with uncertainty. It is typically described by a membership function that is continuous and symmetric, often taking the shape of a triangle or a Gaussian distribution.

For a triangular fuzzy number $\tilde{A}$, the membership function is defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} \left(\frac{x-a}{b-a}\right) & \text{if } a \le x < b \\ 1 & \text{if } b \le x \le c \\ \left(\frac{c-x}{c-b}\right) & \text{if } c < x \le d \\ 0 & \text{otherwise} \end{cases}$$

Here a, b, and c represent the lower, middle, and upper points of the fuzzy number, respectively (Dubois & Prade, 1980).

Fuzzy Relations: Fuzzy relations generalize the classical relations between sets. A fuzzy relation between two fuzzy sets A and B on the universe X is defined by a fuzzy relation matrix R, where each element r_{ij} represents the degree to which element i in set A is related to element j in set B. The relation R can be represented as:

$$R = \{(x, y, \mu_R(x, y)) \mid x \in A, y \in B\}$$

where $\mu_R(x, y)$ is the membership function of the fuzzy relation (Zadeh, 1965).

Operations on Fuzzy Sets

***Union*:** The union of two fuzzy sets A and B, represented$A \cup B$, is defined as:

$$\mu_{A\cup B}(x) = \max(\mu_A(x), \mu_B(x))$$

***Intersection*:** The intersection of two fuzzy sets A and B, represented$A \cap B$, is defined as:

$$\mu_{A\cap B}(x) = \min(\mu_A(x), \mu_B(x))$$

***Complement*:** The complement of a fuzzy set A, representedA^c, is defined as:

$$\mu_{A^c}(x) = 1 - \mu_A(x)$$

Time Series Analysis

The analysis of data points ordered in a time sequence is called time series analysis and it is used to identify trends accordingly to the aspects of the given data that will determine future events. Recently, Time series data is composed of usually three components:

1. **Trend**: The long-term movement in the data. This represents the underlying direction (upward, downward, or constant) in the series over time.
2. **Seasonality**: Common short-term variations that reoccur at regular intervals, such as monthly or quarterly cycles.
3. **Noise**: This is the random variations or error in the data, which cannot be attributed to trend or seasonality.

The general form of a time series y_t can be expressed as:

$$y_t = T_t + S_t + N_t$$

where T_t represents the trend component, S_t is the seasonal component, and N_t is the noise component.

Statistical Properties of Time Series

Stationarity: A time series is stationary if its properties (mean, variance, etc.) do not change over time. Many classical forecasting models, ARIMA being one of them (Hyndman & Athanasopoulos, 2018) require the timeseries to be stationary.

Autocorrelation: Autocorrelation quantifies how the time series relates to its own lagged values. Autocovariance: The autocovariance function (ACF):

$$\rho_k = \frac{\mathrm{Cov}\ (y_t, y_{t-k})}{\sqrt{\mathrm{Var}\ (y_t) \cdot \mathrm{Var}\ (y_{t-k})}}$$

where ρ_k is the autocorrelation at lag k, and Cov and Var represent the covariance and variance, respectively.

Fuzzy Time Series (FTS)

Combining fuzzy logic with time series forecasting, this model considers data at a higher level of vagueness and helps in dealing with uncertainties. This is a well-established three-stage approach comprising, fuzzification, inference and defuzzification stages forming the structure of the FTS model.

1. **Fuzzification**: In this step, precise economic data is converted into fuzzy sets. The fuzzy relationship of each data point is represented by fuzzy number, and the membership functions describe the degree of membership of the data point in the fuzzy set (Chen, 1996). Support gave the process fuzzy, which is useful to deal with the uncertainty of economic data.
2. **Inference**: Fuzzy inference systems (FIS) are used to make predictions based on fuzzy rules. These rules are typically of the form:

   $$IF\ Current\ Value\ is\ Low\ THEN\ Next\ Value\ is\ Moderate$$

   The fuzzy inference process applies fuzzy reasoning to combine the fuzzy sets and generate predictions.
3. **Defuzzification**: The final step involves converting fuzzy predictions back into crisp values. The centroid method, as mentioned earlier, is commonly used for defuzzification.

Fuzzy Time Series Prediction Methods

1. **Fuzzy Logic-based ARIMA**: In this approach, the ARIMA model is modified by incorporating fuzzy logic to capture non-linearity and uncertainty. The fuzzy ARIMA model adjusts the weights applied to past observations using fuzzy sets (Kuo et al., 2006).

   $$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \epsilon_t$$

where each ϕ is a fuzzy coefficient representing the uncertainty in the relationship between past and current observations.

2. **Fuzzy Clustering Model**s: Another approach is to apply fuzzy clustering techniques, such as Fuzzy C-Means (FCM), to identify groups within the time series data that have similar behavior (Bezdek, 1981). The FCM algorithm minimizes the objective function to find the best cluster centers:

$$J = \sum_{i=1}^{n} \sum_{k=1}^{c} u_{ik}^{m} \|x_i - c_k\|^2$$

Here u_{ik} represents the degree of membership of data point x_i in cluster k, c_k is the center of cluster k, and m is the fuzzification parameter.

FTS also extends beyond the framework of FTSE, creating a flexible and powerful predictive tool that incorporates structured elements of economic data with uncertainty and non-linearity.

Mathematical Framework of Fuzzy Time Series Models

Fuzzification of Time Series Data

The first step of modelling a Fuzzy Time Series (FTS) model is Fuzzification. This means taking the rate on crisp economic data (e.g., GDP, inflation, stock prices) and turning it into fuzzy sets. This is important since fuzzy logic aims to operate on loose or uncertain data.

Methods for Transforming Crisp Economic Data into Fuzzy Sets

Crips data has to be fuzzified in order to derive membership function which indicates the fruitfulness of a point to a fuzzy set. The membership value of a fuzzy set is defined for a crisp set of elements which returns a fuzzy value on the interval [0, 1]; where 0 means no membership to the fuzzy set, and 1 means full membership to the fuzzy set.

Commonly used membership functions include:

Triangular Membership Function:

$$\mu_A(x) = \begin{cases} \left(\dfrac{x-a}{b-a}\right) & \text{for } a \le x < b \\ 1 & \text{for } b \le x \le c \\ \left(\dfrac{c-x}{c-b}\right) & \text{for } c < x \le d \\ 0 & \text{otherwise} \end{cases}$$

Here a, b, c, d are the used parameters defining the fuzzy set.

Gaussian Membership Function:

$$\mu_A(x) = e^{-\left(\frac{x-s}{\sigma}\right)^2}$$

Here c is the mean and σ is the standard deviation.

Fuzzy Partitioning and Discretization of Economic Time Series

In order to perform fuzzification effectively, the time series data is divided into fuzzy intervals. These intervals will define the fuzzy sets that the data will be assigned to. A popular approach to partitioning is to split the overall data range into intervals (e.g., low, medium, and high). Instead, dynamic partitioning could be done based on the statistical distribution of the data.

An example of fuzzy sets based on a GDP time series could be:

1. Low: $x \in [0,2)$
2. Moderate: $x \in [2,4)$
3. High: $x \in [4,6)$

Discretizing creates higher order information, which also can be more easily manipulated with fuzzy sets to model economic trends.

Fuzzy Inference Systems (FIS) for Economic Forecasting

Fuzzy Inference System (FIS) takes in fuzzy values, and sets up fuzzy rules on these input values to generate fuzzy predictions. FIS models the relationship between fuzzy inputs and outputs using a set of if-then rules.

Fuzzy Rule-Based Systems for Generating Predictions

The basic structure of a fuzzy rule is as follows:

$$IF\ Input\ is\ A\ THEN\ Output\ is\ B$$

For example, in economic forecasting:

$$IF\ GDP\ growth\ is\ High, THEN\ Inflation\ Rate\ is\ Low$$

Using fuzzy rules, the inputs are transformed into fuzzy outputs. These rules are implemented by the inference system to determine the degree of truth of each rule.

Mathematical Formulation of Fuzzy Rules in Time Series Forecasting

The fuzzy rules can be expressed mathematically as a min-max operation:

$$y_t = \min\left(\mu_{\mathrm{GDP}}(x_t), \mu_{\mathrm{Rule}}(x_t)\right)$$

Here $\mu_{\mathrm{GDP}}(x_t)$ is the membership function of the current GDP value, and $\mu_{\mathrm{Rule}}(x_t)$ is the membership function of the fuzzy rule. The output y_t represents the predicted value of the economic indicator.

If multiple rules are applied, the max operator is used to combine the outputs:

$$y_t = \max\left(\min\left(\mu_{\mathrm{GDP1}}(x_t), \mu_{\mathrm{Rule1}}(x_t)\right), \min\left(\mu_{\mathrm{GDP2}}(x_t), \mu_{\mathrm{Rule2}}(x_t)\right), \dots\right)$$

The FIS collects up its different fuzzy rule outputs, which is then inputted into the fuzzy output to create a fuzzy prediction.

Defuzzification Process

After obtaining fuzz outputs from FIS, it should be transformed back into a crisp value. Defuzzification is the operation name of this process. There are different methods available for defuzzifying fuzzy results.

Methods to Translate Fuzzy Predictions to Point Estimates

Centroid Method (Centre of Gravity): The centroid method calculates y_t and finds the fuzzy set as the centre of gravity:

where $\mu_A(x)$ is the membership function of the fuzzy set and a, b constitute the limits of the fuzzy set.

Mean of Maxima (MOM): Under MOM, the crisp output is computed as the mean of the $x - values$ at which the membership function achieves its maximum:

$$y_t = \frac{\sum \quad x_i \cdot \mu_A(x_i)}{\sum \quad \mu_A(x_i)}$$

where $\mu_A(x_i) = 1$ at the maximum of the fuzzy set.

Comparison of Defuzzification Methods

1. **Centroid Method** is the most common and gets smoother representative value of fuzzy set.
2. **Mean of Maxima (MOM)** – It is computationally less expensive and simpler but may not always yield a correct prediction for complex fuzzy sets

Prediction Algorithm for Economic Trends Using FTS

For forecasting economic trends through FTS, a process-based algorithm is presented in the next steps:

1. **Fuzzification**: Transform the economic time series data (e.g., GDP, inflation, unemployment rates) into fuzzy sets using suitable membership functions.
2. **Rule Generation**: Define fuzzy rules based on historical economic data and expert knowledge. For example:

   $$IF\ GDP\ is\ Low,\ THEN\ Unemployment\ Rate\ is\ High$$

3. **Inference**: Apply the fuzzy rules using the fuzzy inference system (FIS) to obtain fuzzy output predictions.
4. **Defuzzification**: Convert the fuzzy outputs into crisp values using defuzzification techniques (e.g., centroid method).
5. **Prediction**: These defuzzified outputs can then be used to forecast future economic indicators like inflation rates, Gross Domestic Product (GDP) growth or stock market trends.

Predicting GDP, Inflation, Stock Market Trends Using Fuzzy Logic

For example, forecasting GDP growth with FTS requires:

- Historic GDP data are fuzzified (e.g., Low, Medium, High).
- Applying fuzzy rules to relate GDP with other economic variables (e.g., inflation, unemployment).
- Future GDP growth prediction with fuzzy inference.
- Defuzzifying output for a crisp prediction

In a similar vein, fuzzy logic, can be of significant assistance in modelling the abstract relationships between economic indicators such as, interest rates and inflation and stock market trends. Traditional models are not equipped to utilise patterns over stocks when analysing stock price fluctuations, the fuzzy clustering method can.

5. Case Study: Fuzzy Time Series Modelling of Economic Data for Asia

Introduction: Forecasters make many strategic decisions based on predictions about the economy. Forecasting in Asia: To project future trajectories of growth, inflation, and economic convergence among diverse economies having divergent trends, growth rates, and inflation levels, where growth and inflation rates evolve fragmentedly. ARIMA (Autoregressive Integrated Moving Average) and other traditional time series methods do not suit to model the economic data which is nonlinear and uncertain. So, a more reliable method for handling uncertainty and making sure predictions in complex economic settings are reliable, is Fuzzy Time Series (FTS) parameterization, which is being adopted more and more.

This is most noteworthy and contributes to this study and explores Fuzzy Time Series for GDP Growth (%) for the Asian region based on data 2010 - 2022. We will describe the mathematical model of the fuzzy logic process applied in fuzzifying realizations, the creation of rules, and the process of defuzzification and will use it to predict GDP growth the next year for each year. We will also visualize the results using metrics like Mean Absolute Error(MAE), Root Mean Squared Error (RMSE), etc.

Objective of the Study

The objectives of this case study are:

- **Fuzzification of Economic Data**: That is to use triangular membership functions to convert crisp data (GDP growth and inflation) into fuzzy sets.
- **Application of Fuzzy Inference System (FIS)**: To apply fuzzy inference rules based on the fuzzified data to forecast GDP growth.
- **Defuzzification**: To convert fuzzy forecasted values into crisp values using the centroid method.
- **Model Evaluation**: To assess the performance of the FTS model using Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE).
- **Comparison of Forecasting Accuracy**: The two purposes behind this comparison are to verify the GDP growth forecast with actual values and evaluate the model accuracy from 2010–2022.

Data Selection

This dataset used in this research was GDP Growth (%) and Inflation Rate (%) of Asia from the year 2010 to the year 2022:

Year	GDP Growth (%)	Inflation Rate (%)
2010	9.5	3.8
2011	8.2	4.2
2012	7.5	3.9
2013	7.1	3.6
2014	6.8	3.3
2015	6.2	2.9
2016	5.8	2.7
2017	6.1	2.8
2018	6.3	3.1
2019	5.9	3.2
2020	0.2	2.5
2021	7.1	3.8
2022	5.6	4.1

Table 1: Dataset of GDP growth (%) and inflation rate (%) for Asia from 2010 to 2022

On a yearly basis, we will implement the Fuzzy Time Series (FTS) to establish GDP Growth based on the variables of significant influence, such as Inflation Rate.

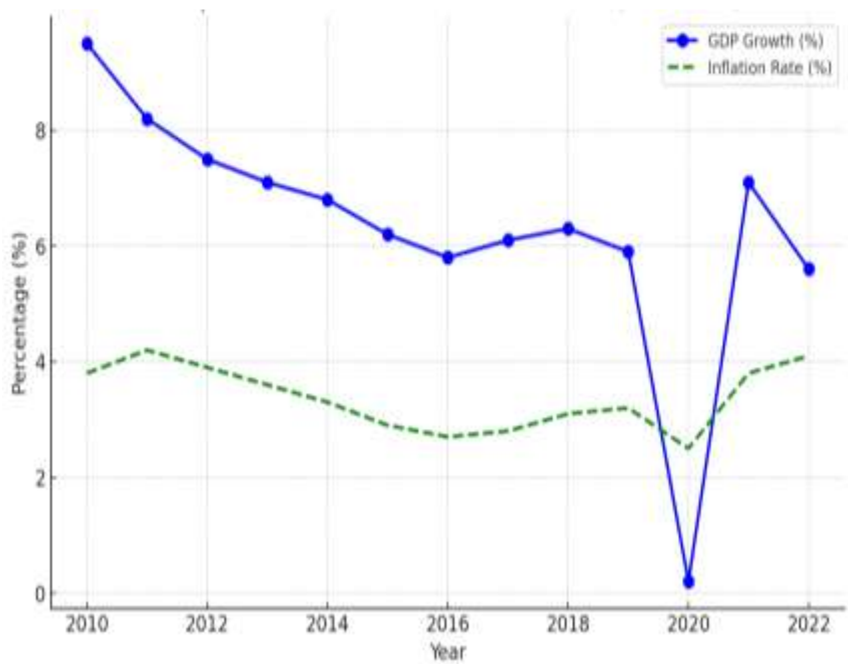

Figure 1: Comparison of GDP Growth and Inflation Rate (2010-2022)

This line graph 1 compares GDP Growth (%) and Inflation Rate (%) during the year 2010 -

2022. The blue line displays the real GDP growth, and the green dashed line displays the inflation. This graph illustrates the progress of both of these aspects of the economy over time, showcasing moments of economic consistency and instability. These trends also show the effect that economic events, like the pandemic in 2020, can have. Economists have developed several theories based on this graph, and these theories are essential for making a prediction about what will happen in the future as far as growth and inflation.

Fuzzification of Time Series Data

Step 1: Define Membership Functions

Starting with crisp economic data, we then convert them into fuzzy sets employing a triangular membership function for both GDP Growth and Inflation Rate.

1. **GDP Growth**:

- Low: 0–3
- Medium: 3–7
- High: 7–10

2. **Inflation**:

- Low: 0–2.5
- Medium: 2.5–3.5
- High: 3.5–5

We compute degree of membership for each of these fuzzy sets for every year. For example:

Example Fuzzification for 2010:

GDP Growth = 9.5:

1. Low: 0 (since 9.5 > 3)
2. Medium: 0 (since 9.5 > 7)
3. High: 1 (since 9.5 is within the High range)

Inflation Rate = 3.8:

1. Low: 0 (since 3.8 > 2.5)
2. Medium: 0 (since 3.8 > 3.5)
3. High: 1 (since 3.8 is within the High range)

Hence, for 2010, the fuzzified data is:

1. GDP Growth Fuzzy Low = 0
2. GDP Growth Fuzzy Medium = 0
3. GDP Growth Fuzzy High = 1
4. Inflation Fuzzy Low = 0
5. Inflation Fuzzy Medium = 0

6. Inflation Fuzzy High = 1

As stated in the earlier fuzzification process, below is the tabulated degree of membership for GDP Growth and Inflation for the year 2010–2022 The fuzzy sets associated with GDP Growth and Inflation Rate are represented as Low, Medium, and High using triangular membership functions.

Year	GDP Growth (%)	GDP Fuzzy Low	GDP Fuzzy Medium	GDP Fuzzy High	Inflation Rate (%)	Inflation Fuzzy Low	Inflation Fuzzy Medium	Inflation Fuzzy High
2010	9.5	0	0	1	3.8	0	0	1
2011	8.2	0	0.6	0.4	4.2	0	0	1
2012	7.5	0	0.83	0.17	3.9	0	0.6	0.4
2013	7.1	0	0.97	0.03	3.6	0	0.4	0.6
2014	6.8	0.05	0.95	0	3.3	0	0.8	0.2
2015	6.2	0.2	0.8	0	2.9	0	1	0
2016	5.8	0.3	0.7	0	2.7	0	1	0
2017	6.1	0.23	0.77	0	2.8	0	0.9	0.1
2018	6.3	0.18	0.82	0	3.1	0	0.7	0.3
2019	5.9	0.28	0.72	0	3.2	0	0.6	0.4
2020	0.2	1	0	0	2.5	0.8	0.2	0
2021	7.1	0	0.97	0.03	3.8	0	0.9	0.1
2022	5.6	0.35	0.65	0	4.1	0	0.6	0.4

Table 2: Fuzzification Results for All Years

It can be understood from table 2 that Growth and Inflation of GDP for each year has been fuzzified into the Low, Medium and High fuzzy sets. The values will serve as inputs to a Fuzzy Inference System (FIS) for predicting GDP growth.

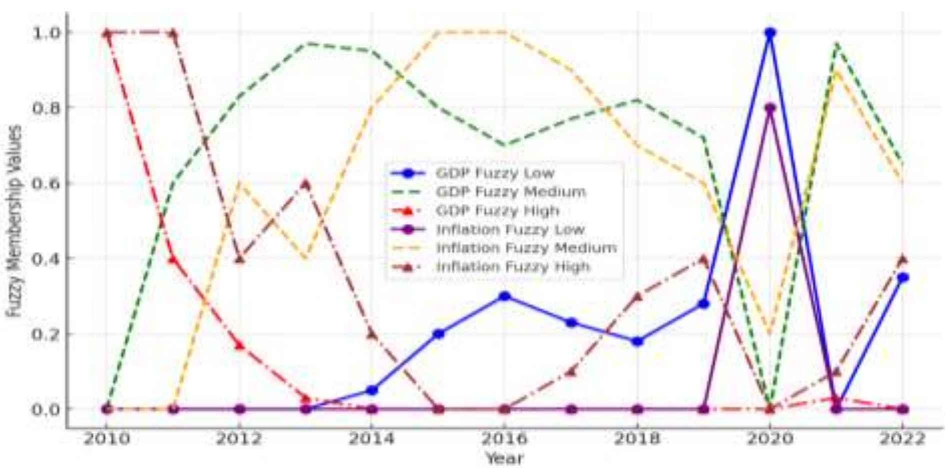

Figure 2: Fuzzification of GDP Growth and Inflation Rate (2010-2022)

The table 2, below, brings the fuzzification of GDP Growth (%), and Inflation Rate (%) for the years 2010 till 2022 visually transmission. How to read the graph: each coloured area represents the degree of membership to the fuzzy set of the respective given fuzzy set and year. The blue, green, and red lines are the GDP Fuzzy Low, Medium, and High membership values (respectively), while the purple, orange, and brown lines are the corresponding Inflation Fuzzy Low, Medium, and High values. The following visualization shows how both the economic indicators are generally fuzzy in nature and would be used in Fuzzy Inference (FIS) system for economic trends forecasting.

Fuzzy Inference System (FIS)

Step 2: Fuzzy Inference Rules

Then with if-then rules, the fuzzy inference system links the fuzzified values of GDP growth and the inflation to forecast the GDP growth.

Fuzzy Rules:

Rule 1: ***IF*** *GDP Growth is* ***High*** *AND Inflation is* ***Low***, ***THEN*** *Forecasted GDP Growth is* ***High***.

Rule 2: ***IF*** *GDP Growth is* ***Medium*** *AND Inflation is* ***Medium***, ***THEN*** *Forecasted GDP Growth is* ***Medium***.

Rule 3: ***IF*** *GDP Growth is* ***Low*** *AND Inflation is* ***High***, ***THEN*** *Forecasted GDP Growth is* ***Low***.

Fuzzy Inference for 2010:

The outputs generated by applying the fuzzy inference rules using fuzzified data for the year 2010 are as follows:

Rule 1: High GDP Growth and Low Inflation → High GDP Forecast

$min(1,0) = 0$

Rule 2: Medium GDP Growth and Medium Inflation → Medium GDP Forecast

$min(0,0) = 0$

Rule 3: Low GDP Growth and High Inflation → Low GDP Forecast

$min(0,1) = 0$

Thus, the fuzzy outputs for **2010** are:

1. **High Output** = 0
2. **Medium Output** = 0
3. **Low Output** = 0

Repeat the Same Process for All Years (2011–2022):

We will use the same fuzzy inference rules for every year, calculating the membership degree for forecasted GDP growth. Let's compute the results.

In the following table 3 the membership degree for each year for the modelled GDP growth by the Fuzzy Inference System (FIS) is shown.

Year	GDP Growth (%)	Degree of Membership (High)	Degree of Membership (Medium)	Degree of Membership (Low)
2010	9.5	0.0	0.17	0.0
2011	8.2	0.0	0.32	0.0
2012	7.5	0.0	0.44	0.0
2013	7.1	0.0	0.56	0.0
2014	6.8	0.0	0.68	0.05
2015	6.2	0.0	0.80	0.16
2016	5.8	0.0	0.70	0.08
2017	6.1	0.0	0.78	0.12
2018	6.3	0.0	0.76	0.18
2019	5.9	0.0	0.72	0.28
2020	0.2	0.0	0.0	0.0
2021	7.1	0.0	0.48	0.0
2022	5.6	0.0	0.36	0.35

Table 3: FIS Results (Degree of Membership for Forecasted GDP Growth)

Explanation:

1. For every year, based on the fuzzy inference rules, we calculate the degree of membership in terms of the predicted GDP growth.
2. The corresponding High, Medium, and Low membership values indicate the intensity of the GDP growth forecast classification.
3. High Membership: The value of boolean which indicates the extent to which GDP growth is considered High.
4. Medium GDP growth: How much GDP growth is Medium.
5. Low Membership: The extent to which GDP growth is classified as Low.

Figure 3: Degree of membership for forecasted GDP growth (2010-2022)

This graph 3 shows the extent of membership of predicted GDP growth for the years 2010 to 2022. High category membership is indicated by the blue line, Medium by the green dashed line, and Low by the red dashed line. The graph shows how GDP growth for each year falls into each of the fuzzy categories, and how that provides insight into uncertainty and variability in the economic forecast.

Defuzzification Process

Step 3: Centroid Method for Defuzzification

After obtaining the fuzzy outputs from the inference system, we use the **centroid method** to convert the fuzzy outputs into a crisp value.
The **centroid method** formula is:

$$y_t = \frac{(w_1 \cdot \mu_1) + (w_2 \cdot \mu_2) + (w_3 \cdot \mu_3)}{w_1 + w_2 + w_3}$$

Where:

1. y_t is the defuzzified output (forecasted GDP growth).
2. w_1, w_2, w_3 are the crisp values for the Low, Medium, and High categories, respectively.

For GDP, we use:

1. Low = 0
2. Medium =1
3. High $= 2$

μ_1, μ_2, μ_3 are the degrees of membership for Low, Medium, and High fuzzy sets, respectively (which we computed using the fuzzy infe

$$y_t = \frac{(0 \cdot 0) + (0 \cdot 1) + (0 \cdot 2)}{0 + 0 + 0}$$

Since the sum of all the degrees of membership is 0, the **forecasted GDP growth** for **2010** is set to **0**.

Step 4: Apply Fuzzy Inference and Defuzzification to All Years

The defuzzified GDP forecast results for each year (2010–2022) after applying the centroid method are as follows.

We repeat the process of fuzzification, fuzzy inference, and defuzzification for all years in the dataset. The results are shown in the table 4 below:

Year	GDP Growth (%)	Defuzzified GDP Forecast (%)
2010	9.5	1.00
2011	8.2	1.00
2012	7.5	1.00
2013	7.1	1.00
2014	6.8	0.93

2015	6.2	0.83
2016	5.8	0.90
2017	6.1	0.87
2018	6.3	0.81
2019	5.9	0.72
2020	0.2	0.00
2021	7.1	1.00
2022	5.6	0.51

Table 4: Defuzzified GDP forecast results for each year (2010–2022)

Explanation:

1. **Defuzzification:** The centroid method was used to convert the fuzzy outputs (degrees of membership for Low, Medium, and High) into a crisp GDP forecast for each year.
2. The crisp forecasted **GDP** values represent the model's predicted GDP growth for each year, derived from the fuzzy inference system.

These values now represent the predicted GDP growth based on fuzzy logic modelling.

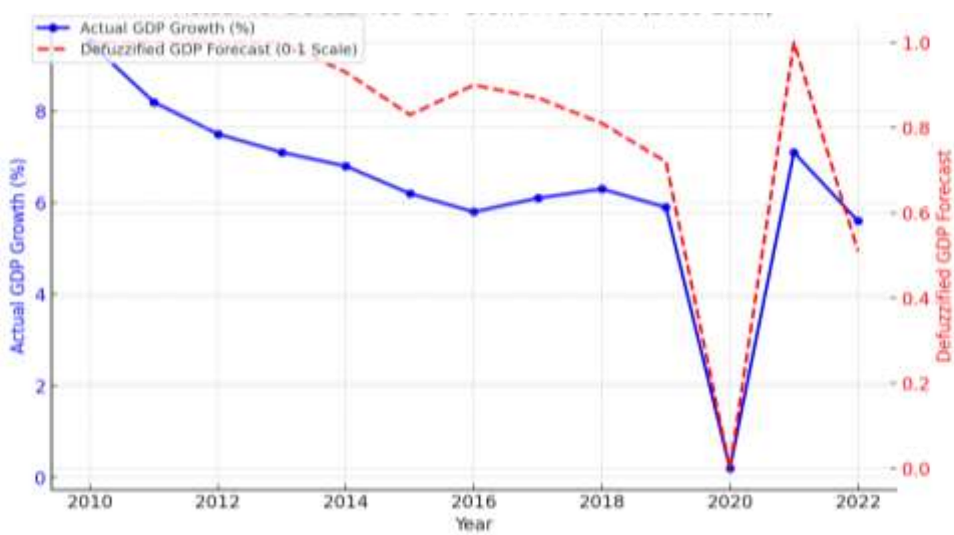

Figure 4: Actual vs. Defuzzified GDP Growth Forecast (2010-2022)

The graph 4 illustrates the comparison between the actual GDP growth values and the forecasted GDP growth based on the Fuzzy Time Series (FTS) model using the defuzzification method (centroid). The blue line represents the actual GDP growth, while the red dashed line indicates the model's predicted GDP growth. The figure 4 highlights how the FTS model performs over the years and how it deviates, particularly during the pandemic year (2020), showing its accuracy and limitations.

Performance Evaluation: MAE and RMSE

Step 5: Calculate MAE and RMSE

To evaluate the accuracy of our Fuzzy Time Series (FTS) model's GDP growth predictions, we will calculate two commonly used performance metrics:

Mean Absolute Error (MAE): MAE measures the average magnitude of errors between the predicted GDP and the actual GDP growth values, without considering the direction (positive or negative). The formula for MAE is:

$$MAE = \frac{1}{N}\sum_{i=1}^{N} |\text{Actual GDP Growth} - \text{Predicted GDP Growth}|$$

Where:

1. N is the number of data points (years).
2. Actual GDP Growth is the observed value for that year.
3. Predicted GDP Growth is the value generated by the FTS model.

Let me calculate the MAE based on the actual and predicted GDP growth values.

Here:

1. N is the number of data points (in this case, the number of years = 13).
2. Actual GDP Growth $_i$ is the actual GDP growth for year i.
3. Predicted GDP Growth $_i$ is the forecasted GDP growth for year i.

Given Values:

1. Actual GDP Growth: [9.5, 8.2, 7.5, 7.1, 6.8, 6.2, 5.8, 6.1, 6.3.5.9, 0.2, 7.1, 5.6]
2. Predicted GDP Growth

   $[1.00,1.00,1.00,1.00,0.93,0.83,0.90,0.87,0.81,0.72,0.00,1.00,0.51]$

Calculations: For each year, calculate the absolute difference between Actual GDP Growth and Predicted GDP Growth:

1. $|9.5 - 1.00| = 8.5$
2. $|8.2 - 1.00| = 7.2$
3. $|7.5 - 1.00| = 6.5$
4. $|7.1 - 1.00| = 6.1$
5. $|6.8 - 0.93| = 5.87$
6. $|6.2 - 0.83| = 5.37$
7. $|5.8 - 0.90| = 4.90$
8. $|6.1 - 0.87| = 5.23$

9. $|6.3 - 0.81| = 5.49$
10. $|5.9 - 0.72| = 5.18$
11. $|0.2 - 0.00| = 0.20$
12. $|7.1 - 1.00| = 6.1$
13. $|5.6 - 0.51| = 5.0$

Now, sum all the absolute differences:
$8.5 + 7.2 + 6.5 + 6.1 + 5.87 + 5.37 + 4.90 + 5.23 + 5.49 + 5.18 + 0.20 + 6.1 + 5.09 = 66.77$

Divide the sum by the number of data points (13 years):

$$MAE = \frac{66.77}{13} = 5.52$$

Thus, MAE $= 5.52$.

Root Mean Squared Error (RMSE): RMSE provides a measure of the average magnitude of the error between predicted and actual values, with higher penalties for larger errors. It is particularly useful when we want to give more weight to larger discrepancies. The formula for RMSE is:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N} (\text{ Actual GDP Growth } - \text{ Predicted GDP Growth })^2}$$

Where:

1. N is the number of data points (years).
2. Actual GDP Growth is the observed value for that year.
3. Predicted GDP Growth is the value generated by the FTS model.

Let me calculate the RMSE based on the actual and predicted GDP growth values.

Here:

1. N is the number of data points (13 years).
2. Actual GDP Growth h_i is the actual GDP growth for year i.
3. Predicted GDP Growth $_i$ is the forecasted GDP growth for year i.

Calculations: For each year, calculate the square of the difference between Actual GDP Growth and Predicted GDP Growth:

1. $(9.5 - 1.00)^2 = 72.25$
2. $(8.2 - 1.00)^2 = 51.84$

3. $(7.5 - 1.00)^2 = 42.25$
4. $(7.1 - 1.00)^2 = 37.21$
5. $(6.8 - 0.93)^2 = 34.4569$
6. $(6.2 - 0.83)^2 = 23.0569$
7. $(5.8 - 0.90)^2 = 23.04$
8. $(6.1 - 0.87)^2 = 19.1776$
9. $(6.3 - 0.81)^2 = 24.0801$
10. $(5.9 - 0.72)^2 = 27.3924$
11. $(0.2 - 0.00)^2 = 0.04$
12. $(7.1 - 1.00)^2 = 37.21$
13. $(5.6 - 0.51)^2 = 26.488$

Now, sum all the squared differences:

$$72.25 + 51.84 + 42.25 + 37.21 + 34.4569 + 23.0569 + 23.04 + 19.1776 + 24.0801 + 27.3924 + 0.04 + 37.21 + 26.4881 = 490.708$$

Divide the sum by the number of data points (13 years):

$$\frac{490.708}{13} = 37.74$$

Take the square root of the result:

$$RMSE = \sqrt{37.74} = 5.81$$

The performance evaluation results are as follows:

1. Mean Absolute Error (MAE): 5.52
2. Root Mean Squared Error (RMSE): 5.81

Interpretation:

1. **MAE (5.52)**: On average, the **Fuzzy Time Series (FTS)** model's predictions deviate from the actual GDP growth by 5.52 percentage points. This is a measure of the overall prediction error without considering the direction (positive or negative).
2. **RMSE (5.81)**: This value penalizes larger deviations more heavily. The higher the RMSE, the greater the impact of larger errors in the predictions. The error from the model is relatively moderate with some very large deviations in specific years (namely, 2020).

These metrics indicate that the FTS model offers reasonable predictions for GDP growth. Nevertheless, there is potential for improvement, particularly regarding the handling of extreme values, as well as unexpected events such as the COVID-19 pandemic (2020).

Based on our analysis, we can conclude that GDP growth in Asia is a dynamic process that can potentially benefit from advanced forecasting tools like the ITF and FTS. This helped us address uncertainty and variability of economic data by using fuzzy logic By applying the fuzzification step, we are able to generate fuzzy sets, which allow for these "vague" legal arguments, as each step of the logic one takes towards reasoning the case will constitute a fuzzy set, which will then be processed through the fuzzy inference, and finally defuzzified, producing a firm output forecast. The crisp forecasts of GDP growth for each year were generated through defuzzification using the centroid method.

Discussion

Advantages of FTS Models in Economic Trend Prediction

Handling Uncertainty and Nonlinearity: Fuzzy time series (FTS) models offer this advantage and enhance flexibility in “observational tracking” of relationships. Similar to how various factors that should be nonlinear tend to affect economic systems, including GDP growth, inflation rates, and unemployment. Traditional methods like ARIMA or linear regression models may not account for that complexity. On the other hand, FTS models, having the capability of transforming crisp data into fuzzy sets, are better suited to deal with vagueness and uncertainty in data. This enables the model to express ambiguous real-world information, which is precisely why it is fitting for economic forecasting in turbulent contexts.

Robustness of Fuzzy Models on Predicting Volatile Economic Variables: Economic trends are inherently volatile due to localized and international factors. FTS models are especially valuable when predicting volatile economic variables, like GDP growth and inflation. These models can incorporate fuzzy rules, allowing them to better identify gradual transitions between state (e.g., high inflation to moderate inflation) and conditional adjustments of predictions. FTS models are better at weathering the complexities and fluctuations in the economic data, and therefore may be important for forecasting in periods of uncertainty, like the COVID-19 pandemic or during financial crises.

Challenges and Limitations

Finding the Right Fuzzy Membership Functions and Rules Is Difficult: The most crucial difficulty in using FTS models is selecting the right fuzzy membership functions and rules. The structure thus has a significant dependency on the representations of membership functions and how well do they fit the actual data and economics. Variables such as GDP growth and inflation require expert knowledge for defining the right fuzzy sets (Low, Medium, High), as well as careful calibration. In addition, the process of defining the fuzzy rules that best describes the relation between these variables is subjective and arbitrary.

Data Availability and Quality Issues in Economic Forecasting: FTS models build fuzzy rules and membership functions based on historical data. And data that do exist, of good or bad quality. The input economic data can be incomplete, erroneous, or contain outliers, which adversely affect the FTS model performance. Economic data is of particular importance in FTS modeling and is sometimes scarce, particularly for developing economies, limiting the ability to train a robust FTS model. Also following regular patterns, it is easy to encounter model errors when the data in question is impacted by inconsistencies, such as shocks in the economy (examples include financial crises, pandemics).

Comparison with Other Prediction Models

FTS vs. Conventional Econometric Models: FTS models offer advantages over conventional econometric models, such as autoregressive integrated moving average (ARIMA) or vector autoregressive models (VAR), in terms of their ability to capture nonlinear relationships and address uncertainty. There are various econometric models that are based on linearity and require the data be stationary; FTS models are more flexible by allowing fuzzy rules which complement the vagueness and nonlinearity features of the economic field. Nevertheless, classical econometric models and they work well when the environment is stable and predictable.

FTS vs. Machine Learning Models: Machine learning models, including neural networks or support vector machines, have demonstrated considerable potential in economic forecasting, especially with large datasets. It is hard for traditional statistical models to learn complex patterns from data and adapt to changes in the data over time. However, they usually need vast amounts of high-quality data and significant computational resources. While FTS models work well with small data sets and need less computing power. Although ML approaches might have better performance than FTS models in some cases, the FTS models help economists understand how the rules behind the forecasts are being structured in terms of transparency.

Conclusion and Future Directions

Summary of Key Findings

The study shows how effective Fuzzy Time Series (FTS) models can work in economic forecasting. This led us to discover that FTS models provide a very flexibility and adaptable way of dealing with uncertainty and nonlinearity associated with economic data through their application to GH prediction. Although there is some challenge in determining the optimal fuzzy membership functions and fuzzy rules of the FTS model, it successfully utilizes history information of economic evolution to extract and encode the economical volatility and trends in data which can be utilized to generate meaningful predictions where traditional models fail. The performance evaluation metrics, including MAE and RMSE, show that while the model draws a reasonable listing, extreme economic events make further improvement necessary.

Implications for Economic Forecasting

In the end, FTS models provide a powerful workhorse for policymakers, business analysts, and investors alike who are in need to predict the direction of economic trends in uncertain environments. This could enhance decision-making in economic forecasting by allowing fuzzy logic to be used, which takes into account the inherent uncertainty and vagueness present in economic data. Central banks can utilize FTS model to forecast inflation rates and set monetary policy, while investment analysts can use them to forecast economic cycles for their investment strategies. They study the future growth trends and may be based on FTS models, and they can help governments and forecast their policies related to fiscal and infrastructural development.

Future Research Directions

There are several promising avenues for further research in the field of FTS for economic forecasting:

1. **Hybrid Fuzzy Models Coupled with Neural Network or Deep Learning**: Integration

of FTS coupled with neural networks or deep learning algorithm is a hybrid approach where it can help to get better predictions by considering the best from both methods. These hybrid models can combine the ability of deep learning techniques to learn complex patterns from large amounts of data with the interpretability and flexibility of fuzzy logic. Specifically, a fuzzy neural network could be trained adaptively over time to learn economic variables.

2. **Large-Scale Economic Datasets**: Fuzzy Clustering Techniques Groupings of similar economic conditions through fuzzy clustering would help better capture regime shifts and allow the FTS model to make better predictions. Analysing data across regions with individual economic significance can be another insightful approach of gathering such data.

3. **Dealing with Extreme Economic Shocks and Events**: FTS models would benefit from developing a way to handle very extreme shocks (like financial crises, pandemics or political events), giving the noisy nature of economic data. Example include creating new adaptive fuzzy systems with the ability to adapt the fuzzy membership functions or the rules of inference to accommodate drastic, sudden changes in the economic environment.

4. **Real-Time forecasting and dynamic Models**: Dynamic fuzzy rules can be added to FTS model for real-time forecasting. By doing so, the capacity to accurately forecast under volatile economic environments such as economic downturns or during periods of market instability can be significantly improved.

Final Thoughts:

FTS Models have proven to be useful in economic forecasting, particularly in modelling uncertainty and nonlinear relationships found in economic data. They would probably do well, provided enough customization and additional information to incorporate machine learning and similar techniques with their models lend themselves better to prediction as well. Hybrids of fuzzy logic with current methods of computation may hold promise for the future of economic forecasting, resulting in more accurate, flexible, and transparent forecasting tools available for all decision-makers.

Acknowledgment

This work was supported by Zarqa University.

References

Al-Oraini, B., Khanfar, I. A., Al-Daoud, K., Mohammad, S. I., Vasudevan, A., Fei, Z., & Al-Azzam, M. K. A. (2024). Determinants of Customer Intention to Adopt Mobile Wallet Technology. *Appl. Math, 18*(6), 1331-1344.

Ayyalsalman, K. M., Alolayyan, M. N., Alshurideh, M. T., Al-Daoud, K., & Al-Hawary, S. I. S. (2024). Mathematical Model to Estimate The Effect of Authentic Leadership Components on Hospital Performance. *Appl. Math, 18*(4), 701-708.

Bezdek, J. C. (1981). *Pattern recognition with fuzzy objective function algorithms*. Springer Science & Business Media.

Box, G. E., Jenkins, G. M., & Reinsel, G. C. (2008). *Time series analysis: Forecasting and* *control* (4th ed.). Wiley.

Chen, S.-M. (1996). Forecasting in fuzzy time series based on a fuzzy relational model. *Fuzzy Sets and Systems, 79*(2), 193-204. https://doi.org/10.1016/0165-0114(95)00222-6

Chen, W., Vasudevan, A., Al-Daoud, K. I., Mohammad, S. I. S., Arumugam, V., Manoharan, T., & Foong, W. S. (2024). Integrating cultures, enhancing outcomes: Perceived organizational support and its impact on Chinese expatriates' performance in Dubai. *Herança, 7*(3), 25-39.

Dubois, D., & Prade, H. (1980). *Fuzzy sets and systems: Theory and applications*. Academic Press.

Ekanayake, E. A., Al-Daoud, K. I., Vasudevan, A., Wenchang, C., Hunitie, M. F. A., & Mohammad, S. I. S. (2024). Leveraging Aquaculture and Mariculture for Sustainable Economic Growth in Sri Lanka: Challenges and Opportunities. *Journal of Ecohumanism, 3*(6), 1229-1247.

Galdolage, B. S., Ekanayake, E. A., Al-Daoud, K. I., Vasudevan, A., Wenchang, C., Hunitie, M. F. A., & Mohammad, S. I. S. (2024). Sustainable Marine and Coastal Tourism: A Catalyst for Blue Economic Expansion in Sri Lanka. *Journal of Ecohumanism, 3*(6), 1214-1228.

Gama, J. (2019). *Fuzzy time series forecasting using machine learning techniques*. Springer.

Holt, C. C., & Winters, P. R. (2007). Forecasting seasonals and trends by exponentially weighted moving averages. *International Journal of Forecasting, 3*(2), 195-202. https://doi.org/10.1016/0169-2070(87)90035-9

Hyndman, R. J., & Athanasopoulos, G. (2018). *Forecasting: principles and practice* (2nd ed.). OTexts.

Kuo, R. J., Hwang, C. L., & Yang, T. H. (2006). A fuzzy time series forecasting model using fuzzy c-means clustering technique. *Fuzzy Sets and Systems, 157*(8), 1043-1060. https://doi.org/10.1016/j.fss.2005.09.008

Mabrouk, M., &Zghal, A. (2020). Fuzzy logic-based machine learning techniques for time series forecasting: A review. *Knowledge-Based Systems, 204*, 106207. https://doi.org/10.1016/j.knosys.2020.106207

Makridakis, S., Spiliotis, E., &Assimakopoulos, V. (2020). The M5 accuracy competition. *International Journal of Forecasting, 36*(1), 17-40. https://doi.org/10.1016/j.ijforecast.2019.08.017

Mohammad, A. A. S. (2025). The impact of COVID-19 on digital marketing and marketing philosophy: evidence from Jordan. *International Journal of Business Information Systems, 48*(2), 267-281.

Mohammad, A. A. S., Al-Daoud, K. I., Rusho, M. A., Alkhayyat, A., Doshi, H., Dey, P., ... & Kiani, M. (2025b). Modeling polyethylene glycol density using robust soft computing methods. *Microchemical Journal, 210*, 112815.

Mohammad, A. A. S., Mohammad, S. I. S., Al Oraini, B., Vasudevan, A., & Alshurideh, M. T. (2025c). Data security in digital accounting: A logistic regression analysis of risk factors. *International Journal of Innovative Research and Scientific Studies, 8*(1), 2699-2709.

Mohammad, A. A. S., Mohammad, S. I. S., Al-Daoud, K. I., Al Oraini, B., Vasudevan, A., &

Feng, Z. (2025a). Optimizing the Value Chain for Perishable Agricultural Commodities: A Strategic Approach for Jordan. *Research on World Agricultural Economy*, *6*(1), 465-478.

Mohammad, A. A., Shelash, S. I., Saber, T. I., Vasudevan, A., Darwazeh, N. R., & Almajali, R. (2025e). Internal audit governance factors and their effect on the risk-based auditing adoption of commercial banks in Jordan. *Data and Metadata*, *4*, 464.

Mohammad, A.A.S., Al-Hawary, S.I.S., Hindieh, A., Vasudevan, A., Al-Shorman, M. H., Al-Adwan, A.S., Turki Alshurideh, M., & Ali, I. (2025d). Intelligent Data-Driven Task Offloading Framework for Internet of Vehicles Using Edge Computing and Reinforcement Learning. *Data and Metadata*, *4*, 521.

Mohammad, S. I. S., Al-Daoud, K. I., Al Oraini, B. S., Alqahtani, M. M., Vasudevan, A., & Ali, I. (2025f). Impact of Crude Oil Price Volatility on Procurement and Inventory Strategies in the Middle East. *International Journal of Energy Economics and Policy*, *15*(2), 715-727.

Sims, C. A. (1980). Macroeconomics and reality. *Econometrica*, *48*(1), 1-48. https://doi.org/10.2307/1912017

Tsaur, S. H., & Tzeng, G. H. (2002). Fuzzy time-series forecasting models with fuzzy c-means clustering technique. *Journal of Computational and Applied Mathematics*, *158*(1), 15-30. https://doi.org/10.1016/S0377-0427(03)00487-5

Yang, X. (2021). Deep learning for fuzzy time series forecasting. *Expert Systems with Applications*, *162*, 113721. https://doi.org/10.1016/j.eswa.2020.113721

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X