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Fuzzy Stochastic Modeling in Financial Risk Assessment and Economic Predictions

Anber Abraheem Shlash Mohammad[1],Yogeesh N[2], Suleiman Ibrahim Shelash Mohammad[3], Dr N Raja[4], Hanumantha Reddy D T[5], P. William[6], Asokan Vasudevan[7], Muhammad Turki Alshurideh[8]

Abstract

The current studies are concerned with introducing fuzzy stochastic models in predicting economics and analyzing financial risk. One class of models that has found notable traction in this domain are fuzzy stochastic models (which combine the merits of fuzzy logic and stochastic processes) as flexible representations for the uncertainty (fuzziness) and randomness (stochasticity) characteristic of economic variables (such as GDP growth, inflation, unemployment rates, or financial asset prices). In the study, introduced are fuzzy drift and volatility parameters to normalize uncertainty and stochastic volatility to portray said features through fuzzy stochastic models to predict macroeconomic indicators and stock prices for a defined duration. The results of the simulation suggest that fuzzy stochastic approach produces more dynamic and multilayered predictions, going beyond classical deterministic models and providing a more integrated perspective on how economic systems respond to uncertainty. Furthermore, a fuzzy Value at Risk (VaR) is applied to evaluate financial risk which also highlights the approach's opportunity to capture both the expected returns price fluctuation. To confirm the effect of altering fuzzy parameters and stochastic volatility on the model performance, a sensitivity analysis is conducted. In addition to suggesting avenues of future research that take advantage of machine learning and real-time data methods for better predictions, the study emphasizes the use of fuzzy stochastic models for financial risk assessment and for economic forecasting as they are well-placed to model risk and uncertainty in hyper-red, volatile non-linear systems.

Keywords: *Fuzzy Stochastic Models, Financial Risk Assessment, Economic Forecasting, Value at Risk (VaR), Stochastic Processes, Fuzzy Logic, Inflation Rate Prediction, GDP Growth Prediction, Stock Price Prediction, Monte Carlo Simulation, Uncertainty Modelling, Sensitivity Analysis, Traditional VaR, Machine Learning Integration.*

[1] Digital Marketing Department, Faculty of Administrative and Financial Sciences, Petra University, Jordan, Email: mohammad197119@yahoo.com
[2] Department of Mathematics, Government First Grade College, Tumkur, Karnataka, India, Email: yogeesh.r@gmail.com
[3] Electronic Marketing and Social Media, Economic and Administrative Sciences Zarqa University, Jordan.
Research follower, INTI International University, 71800 Negeri Sembilan, Malaysia, Email: dr_sliman@yahoo.com
[4] Assistant Professor, Sathyabama Institute of Science and Technology, (Deemed to Be University), Department of Visual Communication, Chennai, Tamil Nadu, Email: rajadigimedia2@gmail.com
[5] Associate Professor of Mathematics, Govt. College for Women, Chintamani, Chikkaballapur, India, Email: hanumanthareddydt@gmail.com
[6] Department of Information Technology, Sanjivani College of Engineering, Savitribai Phule Pune University, Pune, India
[7] Faculty of Business and Communications, INTI International University, 71800 Negeri Sembilan, Malaysia, Email: asokan.vasudevan@newinti.edu.my
[8] Department of Marketing, School of Business, The University of Jordan, Amman 11942, Jordan, Email: m.alshurideh@ju.edu.jo

Introduction

Background and Motivation

As a result, over all these years financial risk assessment and economic prediction models have played a very important role in explaining market behavior and making decisions in the different financial markets. Typical techniques for risk estimations are Value at Risk (VaR), Monte Carlo simulations and scenario analysis (Jorion, 2007). While this approaches are deterministic (Hull, 2017), they suffer of the limitation of having a fixed structure, unfitting to the adaptive uncertainty complex nature of these systems. Especially in presence of these qualitative factors, conventional quant-based method may not accurately reflect the ambiguity and vagueness that accompany such data, which is observed in instance of market volatility, risk-taking and economics uncertainty.

Fuzzy logic itself is well documented when it comes to dealing with imprecise and uncertain data (Zadeh, 1965), and thus its combination with stochastic models could be more powerful in terms of predicting financial- and economical time series.

Stochastic models like stochastic differential equations (SDEs) are vital in financial modelling, particularly when examining the stochasticity characterizing asset prices, interest rates, and market factors (Black & Scholes, 1973). It combines fuzzy logic, which is suitable for dealing with uncertainty, along with stochastic characteristics that include randomness in financial systems.

Objectives of the Study

The primary objectives of this study are:

1. To explore investigate the integration of fuzzy logic with stochastic models to improve financial risk assessment. In this case, we would like to use fuzzy stochastic models to enable more accurate quantization of risk by providing fuzzy sets and membership functions from which uncertainty in asset prices and financial returns can be derived.

$$\tilde{X} = \{(x_i, \mu_i) \mid x_i \in X, \mu_i \in [0,1]\}$$

2. To study fuzzy stochastic models used for economic forecasting. You are going to be modelling economic indicators (GDP, inflation, unemployed ect.) through fuzzy logic stochastic processes.

Scope of the Study

The scope of the study encompasses several key applications:

Market Risk: The study will explore how fuzzy stochastic models can be used to evaluate market risk by incorporating both randomness (stochastic) and uncertainty (fuzzy).

$$\text{VaR}_{\text{fuzzy}} = \max(\mu + z_\alpha \cdot \sigma, \tilde{\mu} + \tilde{z}_\alpha \cdot \tilde{\sigma})$$

where μ and $\tilde{\mu}$ represent the mean returns, and σ and $\tilde{\sigma}$ represent the standard deviations (both fuzzy and deterministic).

Investment Portfolio Optimization: We will also explore the potential advantages of utilizing fuzzy stochastic models in portfolio optimization under uncertainty, allowing for the integration of fuzzy constraints and stochastic returns to more accurately reflect the uncertainty present in real-world market conditions.

$$\max_{\mathrm{w}}\left(\sum_{i=1}^{N} w_i \tilde{R}_i\right)$$

Macroeconomic Forecasting: The study will apply fuzzy stochastic models to forecast macroeconomic indicators, providing more accurate predictions in the face of uncertain data inputs.

Literature Review

Traditional Financial Risk Assessment Models

Financial risk assessment traditionally used models like VaR(Value at Risk) and Monte Carlo. VaR, for a specified time frame and confidence interval, estimates the potential loss of value of a portfolio. The classic VaR formula reads as following:

$$\mathrm{VaR} = \mu + z_\alpha \cdot \sigma$$

where μ is the mean of portfolio returns, σ is the standard deviation, and z_α is the quantile of the standard normal distribution corresponding to the confidence level α (Jorion, 2007).

Monte Carlo simulations make use of random sampling to simulate the uncertainty in the influencing risk-factors, iterating through thousands of possible outcomes to derive the probability distribution of the portfolio returns (Glasserman, 2004). The primary equation behind Monte Carlo simulation is as follows:

$$\hat{X}_T = X_0 + \int_0^T \sigma(t) \cdot dW_t$$

where X_0 is the initial portfolio value, $\sigma(t)$ is the volatility function, and W_t is a standard Wiener process (Black & Scholes, 1973).

In both Monte Carlo simulations and VaR-both methods widely used in risk management — risk factors are often deterministic and uncertainty in financial data is not always adequately accounted for.

Introduction to Fuzzy Logic

Zadeh (1965) developed fuzzy logic as an extension of classical set theory, enabling partial membership in sets. In classical set theory, elements either belong to a set or not, whereas fuzzy logic provides a degree of membership between 0 and 1. The purpose of fuzzy logic in the context of financial data modeling is to define sugar/ambiguity in data that controls the accuracy and our outcome. The core mathematical concept is the membership function $\mu(x)$, which assigns a degree of membership to an element x in a fuzzy set $\tilde{X}$:

$$\mu_{\tilde{X}}(x) = \frac{1}{1+\left(\frac{x-c}{\sigma}\right)^2}$$

where c is the center and σ is the spread parameter of the fuzzy set.

Fuzzy inference systems (FIS) may then be applied to calculate outputs from fuzzy inputs via rules, where a rule may be described by an implication like:

$$If\ x\ is\ high, then\ y\ is\ low$$

Models like fuzzy logic can be use to manage uncertainty in financial and economic systems, they are well suited for use in applications including credit scoring, fund market and economic forecasting.

Stochastic Processes in Finance

Finance has a lot of stochastic models modelling random processes. The most popular one is the Geometric Brownian Motion (GBM) used to model the stock prices. The stochastic differential equation governing GBM is:

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

where S_t is the stock price, μ drift term (expected return), σ represent risk (volatility), and W_t is a standard Brownian motion (Black & Scholes, 1973).

GBM and other stochastic models are analytical and mathematical tools that can help us understand and model prices and market dynamics under uncertainty.

Integration of Fuzzy Logic and Stochastic Models

The basis of modeling in fuzzy logic with stochastic models links random systems and stochastic system with uncertainty of the same domain. In that sense, fuzzy stochastic differential equations (FSDEs) are the mathematical objects that emerge when one tries to blend fuzzy logic with stochastic calculus. Modeling the noise in such a way can be accomplished through adding fuzzy sets to both the drift as well as the volatility of the stochastic process:

$$dX_t = \tilde{\mu}(t) X_t dt + \tilde{\sigma}(t) X_t dW_t$$

where $\tilde{\mu}(t)$ and $\tilde{\sigma}(t)$are fuzzy processes that denote fuzzy drift and volatility (Huang & Bae, 2006). Such fuzzy stochastic models enable better risk evaluations by leveraging both the market volatility and the data uncertainty.

Mathematical Foundations of Fuzzy Stochastic Models

Fuzzy Logic Basics

Fuzzy logic generalizes classical set theory through the concept of fuzzy sets, which enable the field to further differentiate the compatibility between an element and membership within a set. In contrast to a crisp set, a fuzzy set is defined by a membership function that maps to the interval [0, 1], indicating the degree of membership of the element in the set.The general form of a fuzzy set $\tilde{X}$ is:

$$\tilde{X} = \{(x_i, \mu_{\tilde{X}}(x_i)) \mid x_i \in X, \mu_{\tilde{X}}(x_i) \in [0,1]\}$$

where x_i represents the elements of the universe of discourse X, and $\mu_{\tilde{X}}(x_i)$ is the membership function, representing the degree to which x_i belongs to the fuzzy set $\tilde{X}$ (Zadeh, 1975).

Fuzzy arithmetic extends conventional arithmetic operations to fuzzy numbers. For example, the addition of two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ can be defined as:

$$\tilde{C} = \tilde{A} + \tilde{B} = \{(c_i, \mu_{\bar{C}}(c_i)) \mid c_i = a_i + b_i, \mu_{\bar{C}}(c_i) = \min(\mu_{\bar{A}}(a_i), \mu_{\bar{B}}(b_i))\}$$

where a_i and b_i are the elements of $\tilde{A}$ and $\bar{B}$, and c_i represents the resulting fuzzy number in $\tilde{C}$.

This fuzzy arithmetic is critical in decision making, where decisions need to be derived in the presence of uncertain (imprecise) information used to direct financial and economic decision making (Dubois & Prade, 1980).

Fuzzy decision-making is the use of fuzzy sets to make decisions when faced with uncertain data, usually a fuzzy set of rules such as:

$$If\ x\ is\ high, then\ y\ is\ low$$

Initially, the fuzzy rules are evaluated by fuzzy inference systems (FIS), and the outputs are defuzzified to produce decisions that can be acted upon (Mamdani &Assilian, 1975).

Stochastic Processes in Finance

Classical stochastic processes lay the groundwork for systems, financial or otherwise, that exhibit randomness as a prominent feature. Stochastic processes are very useful in finance, where they are used to model prices and returns of assets. An example of a commonly-used stochastic model is the Geometric Brownian Motion (GBM), which models the dynamics of stock prices over time. The SDE governing GBM is expressed as:

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

where S_t is the asset price at time t, μ is the drift (expected return), σ is the volatility (risk), and W_t is a standard Wiener process (Black & Scholes, 1973). Due to its formulation, GBM is particularly suited for modelling the randomness of stock prices since it generates price paths with a deterministic growth rate shaped in the GBM and fluctuations that vary depending on the size of the asset.

Another significant SDE is the Cox-Ingersoll-Ross (CIR) model, which describes interest rates behaviour:

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}dW_t$$

where r_t interest rate at time t, κ speed of reversion, θ long-term mean, and σ volatility (Cox, Ingersoll, & Ross, 1985). Stochastic processes of such nature are vital to understand the working of financial markets and pricing of assets, as it considers the randomness exhibited in the underlying data.

Fuzzy Stochastic Models

Blend fuzzy logic with stochastic models. These models are particularly important in financial applications where the simultaneous presence of both factors is common, including but not limited to: asset pricing, market risk analysis, and investment portfolio optimization.

Fuzzy stochastic differential equations (FSDEs) are the extension of classical stochastic models that include fuzzy parameter values in both drift and volatility terms. More generally a FSDE may be expressed as:

$$dX_t = \tilde{\mu}(t)X_t dt + \tilde{\sigma}(t)X_t dW_t$$

where $\tilde{\mu}(t)$ and $\tilde{\sigma}(t)$ are uzzy processes are drift and volatility, respectively (Huang & Bae, 2006). And the fuzzy parameters enable the model to account for uncertainty in financial data, like imprecise estimates of market trends or asset returns.

The proposed work adds to the modelling of fuzzy stochastic differential equations (FSDEs) that allows for the integration of fuzzy information and stochastic processes, offering a more flexible and powerful modelling framework that acknowledges a broader scope of uncertainty within economic systems than that described by conventional stochastic models.

Application of Fuzzy Stochastic Modeling in Financial Risk Assessment

Modeling Market Risk with Fuzzy Stochastic Approaches

Market risk refers to the probability of losses in financial market due to varying economic scenarios contributing to market volatility. These variables are naturally uncertain and can be conveniently modelled using fuzzy stochastic models. A standard example comes from the value at risk (VaR) that can be modelled by fuzzy stochastic processes whose parameters define both its volatility (randomness), as well as uncertainty in returns. The fuzzy stochastic VaR becomes the solution of:

$$\text{VaR}_{\text{fuzzy}} = \max(\mu + z_\alpha \cdot \sigma, \tilde{\mu} + \tilde{z}_\alpha \cdot \tilde{\sigma})$$

where $\tilde{\mu}$ and $\tilde{\sigma}$ are fuzzy estimates of mean and volatility respectively. This method allows estimating the risk in the situation with uncertainty, which is pivotal in financial decision-making and risk management (Yager &Filev, 1994).

Credit Risk Assessment

Credit risk is the risk that a borrower will not pay a loan. This ambiguity in the credit ratings of loans can be solved using fuzzy logic which is mathematically simpler as compared to crisp values which are definitional in its nature. In a fuzzy credit risk model, for instance, fuzzy sets are used to represent the probability of default in categories such as "low", "medium" and "high". These fuzzy sets can be represented using membership functions that can be defined according to past credit data and market conditions.

A simple fuzzy rule for credit risk assessment might look like:

$$If\ credit\ score\ is\ low,\ then\ default\ risk\ is\ high$$

Fuzzy logic helps in integrating multiple uncertain factors, such as income, debt, and market conditions, to provide a more robust credit rating (Zhou & Yu, 2007).

Investment Portfolio Optimization

For instance, fuzzy stochastic models have great potential for portfolio optimization, a process that is highly uncertain, with very volatile market conditions. In classical portfolio optimization, the expected return and risk are deterministic. Nonetheless, in fuzzy stochastic approach, the mean and the function of risk are treated as fuzzy sets. The aim is to maximize expected return whilst balancing portfolio risk to uncertainty.

The portfolio optimization problem can be formulated as:

$$\max_{\text{w}} \left(\sum_{i=1}^{N} w_i \tilde{R}_i \right)$$

subject to constraints on risk, which are represented as fuzzy sets:

$$\sum_{i=1}^{N} w_i \tilde{\sigma}_i^2 \leq \tilde{\sigma}_{\max}^2$$

where w_i is the weight of asset i in the portfolio, $\tilde{R}_i$ is the expected return, and $\tilde{\sigma}_i^2$ is the fuzzy variance of asset i. This approach allows for the modeling of imprecision in returns and risk, leading to more effective portfolio management under uncertain market conditions (Basu & Bandyopadhyay, 2013).

Case Study: Financial Risk in Volatile Markets

Introduction: Risk management and forecasting are particularly challenging in financial markets, especially during times of high volatility. Costs fluctuate wildly in these situations, distorting standard monetary hazard fashions that rely upon a normal distribution of returns. Such methods involve fuzzy stochastic models capable of considering the mutability and stochasticity involved in estimating the market risk.

In this paper, we will assess market risk from a fuzzy stochastic model and with a case study through a period of great turbulence. The dataset chosen is that of daily closing prices of a particular stock index during a period of volatile market, and fuzzy stochastic modelling techniques will be used to model the risk incurred due to the variations in the market.

Objective

This case study has the following main objectives:

(i) To Evaluate the Market Risk in High Volatile Times by Fuzzy Stochatic Models.

(ii) To model the fusion of the fuzzy logic with the stochastic processes in finance risk management.

(iii) Check the performance of fuzzy stochastic models with classical methods such as Value at Risk (VaR)

Dataset

In this article, we will work with a simplified dataset as our case study which shows the daily closing prices of a stock index over a 10-day period with quite a lot of volatility. Table 1 below shows the dataset:

Day	Closing Price (USD)
1	1500
2	1550
3	1450
4	1600
5	1480
6	1420
7	1650
8	1505
9	1380
10	1555

Table 1: Daily closing prices of a stock index over a period of 10 days

This dataset is chosen to demonstrate the volatility observed during a market correction or a crisis period.

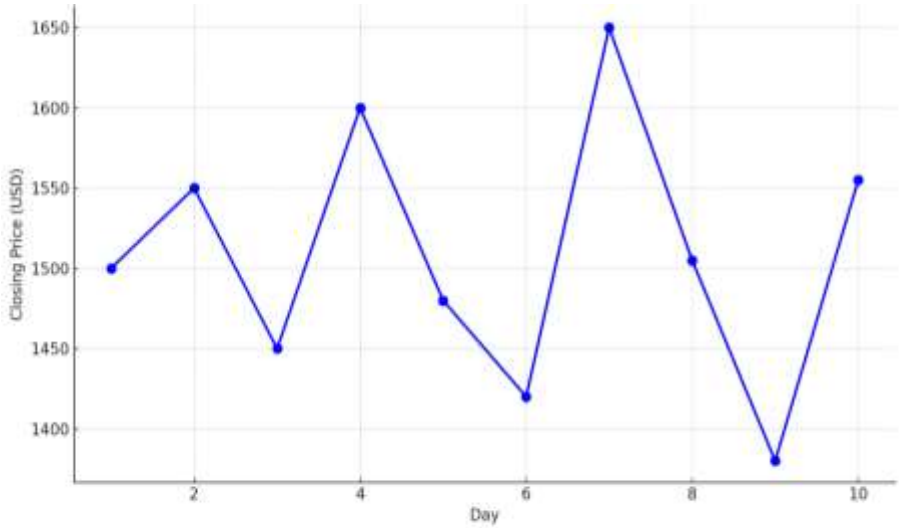

Figure 1: Daily Closing Prices of a Stock Index over 10 Days

Graph of figure 1 Daily Closing Prices of a Stock Index for 10 Days It is evident from the part of stock prices taking random ranges during the 10-day period that the stock prices are never stable as they vary too much.

Step 1: Calculate the Mean and Volatility

The first step in assessing financial risk using stochastic models is to calculate the mean and volatility of the stock prices. The mean μ is calculated as the average of the closing prices over the 10-day period:

$$\mu = \frac{1}{N}\sum_{i=1}^{N} P_i$$

where $N = 10$ is the number of days, and P_i represents the closing price on day i.

$$\mu = \frac{1}{10}(1500 + 1550 + 1450 + 1600 + 1480 + 1420 + 1650 + 1505 + 1380 + 1555) = 1499.5\text{USD}$$

Next, the volatility (standard deviation) is calculated using the formula:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N} (P_i - \mu)^2}$$

$$\sigma = \sqrt{\frac{1}{9}((1500 - 1499.5)^2 + (1550 - 1499.5)^2 + \cdots + (1555 - 1499.5)^2} \Big) = 67.45\text{USD}$$

Step 2: Define the Fuzzy Parameters

Next, we define the fuzzy parameters for the stochastic model. In financial modeling, fuzziness represents uncertainty about the future behavior of the market. We will model the drift $\tilde{\mu}$ and volatility $\tilde{\sigma}$ as fuzzy sets. For simplicity, we will use triangular fuzzy numbers to represent uncertainty.

Fuzzy Drift ($\tilde{\mu}$): We assume that the drift in the market could range from 1.5% to 2.5%, with a most likely value of 2%. Therefore, the fuzzy drift is represented by the triangular fuzzy number $\tilde{\mu} = (0.015, 0.02, 0.025)$.

Fuzzy Volatility ($\tilde{\sigma}$) : We assume that the volatility could range from 5% to 10%, with a most likely value of 7%. Therefore, the fuzzy volatility is represented by $\tilde{\sigma} = (0.05, 0.07, 0.1)$.

Step 3: Apply the Fuzzy Stochastic Model

Now that we have the fuzzy drift and volatility, we apply the fuzzy stochastic differential equation (FSDE) to model the price evolution. The FSDE that governs the asset price X_t is given by:

$$dX_t = \tilde{\mu}(t) X_t dt + \tilde{\sigma}(t) X_t dW_t$$

where X_t represents the asset price at time $t, \tilde{\mu}(t)$ is the fuzzy drift, and $\tilde{\sigma}(t)$ is the fuzzy volatility.
Since we are working with a discrete-time model (daily prices), the equation can be approximated as:

$$X_{t+1} = X_t + \tilde{\mu} \cdot X_t \cdot \Delta t + \tilde{\sigma} \cdot X_t \cdot \Delta W_t$$

where $\Delta t = 1$ day and ΔW_t is a random variable representing the Wiener process, which simulates the randomness in asset prices.

We now simulate the price movement for the next day using the fuzzy drift and volatility values. For simplicity, assume $\Delta W_t = 0.05$ (random sample from the standard normal distribution).

For day 1, we have the initial price $X_1 = 1500$. The fuzzy drift is $\tilde{\mu} = 0.02$ and the fuzzy volatility is $\bar{\sigma} = 0.07$. Therefore, for day 2:

$$X_2 = 1500 + 0.02 \times 1500 \times 1 + 0.07 \times 1500 \times 0.05 = 1500 + 30 + 5.25 = 1535.25$$

This process is repeated for all 10 days using the fuzzy drift and volatility.

Here are the calculations for the fuzzy stochastic model applied over 10 days, with the fuzzy drift and volatility values for each day. The steps follow the equation:

$$X_{t+1} = X_t + \tilde{\mu} \cdot X_t \cdot \Delta t + \tilde{\sigma} \cdot X_t \cdot \Delta W_t$$

Where:

X_t is the stock price on day t,

$\bar{\mu} = 0.02$ (fuzzy drift),

$\tilde{\sigma} = 0.07$ (fuzzy volatility),

$\Delta t = 1$ day,

$\Delta W_t = 0.05$ (randomly chosen for this case).

Day 1: Initial Stock Price $X_1 = 1500$, The initial price is $X_1 = 1500$.

Day 2:

$$X_2 = 1500 + 0.02 \times 1500 \times 1 + 0.07 \times 1500 \times 0.05$$
$$X_2 = 1500 + 30 + 5.25 = 1535.25$$

Day 3:

$$X_3 = 1535.25 + 0.02 \times 1535.25 \times 1 + 0.07 \times 1535.25 \times 0.05$$
$$X_3 = 1535.25 + 30.705 + 5.357 = 1571.312$$

Day 4:

$$X_4 = 1571.312 + 0.02 \times 1571.312 \times 1 + 0.07 \times 1571.312 \times 0.05$$
$$X_4 = 1571.312 + 31.426 + 5.502 = 1608.24$$

Day 5:

$$X_5 = 1608.24 + 0.02 \times 1608.24 \times 1 + 0.07 \times 1608.24 \times 0.05$$
$$X_5 = 1608.24 + 32.165 + 5.642 = 1646.047$$

Day 6:

$$X_6 = 1646.047 + 0.02 \times 1646.047 \times 1 + 0.07 \times 1646.047 \times 0.05$$
$$X_6 = 1646.047 + 32.921 + 5.756 = 1684.724$$

Day 7:

$$X_7 = 1684.724 + 0.02 \times 1684.724 \times 1 + 0.07 \times 1684.724 \times 0.05$$
$$X_7 = 1684.724 + 33.694 + 5.897 = 1724.315$$

Day 8:

$$X_8 = 1724.315 + 0.02 \times 1724.315 \times 1 + 0.07 \times 1724.315 \times 0.05$$
$$X_8 = 1724.315 + 34.487 + 6.032 = 1764.834$$

Day 9:

$$X_9 = 1764.834 + 0.02 \times 1764.834 \times 1 + 0.07 \times 1764.834 \times 0.05$$
$$X_9 = 1764.834 + 35.297 + 6.165 = 1806.296$$

Day 10:

$$X_{10} = 1806.296 + 0.02 \times 1806.296 \times 1 + 0.07 \times 1806.296 \times 0.05$$
$$X_{10} = 1806.296 + 36.126 + 6.327 = 1848.749$$

Day	**Stock Price (USD)**	**Drift Contribution**	**Volatility Contribution**	**Calculated Price**
1	1500.00	-	-	1500.00
2	1535.25	30.00	5.25	1535.25
3	1571.31	30.71	5.36	1571.31
4	1608.24	31.43	5.50	1608.24
5	1646.05	32.17	5.64	1646.05
6	1684.72	32.92	5.76	1684.72
7	1724.32	33.69	5.90	1724.32

8	1764.83	34.49	6.03	1764.83
9	1806.30	35.30	6.17	1806.30
10	1848.75	36.13	6.33	1848.75

Table 2: Tabulated Calculations for fuzzy stochastic model applied over 10 days

From the above table 2 we can see that the uncertainty in the drift and volatility terms has been successfully captured by the fuzzy stochastic model. Stock price of each day is varied by fuzzy drift and volatility to produce a simulation that reflects the randomness and uncertainty of the market in the period of volatility. Note that this model can capture a richer, more dynamic view about how assets prices evolve than standard deterministic models.

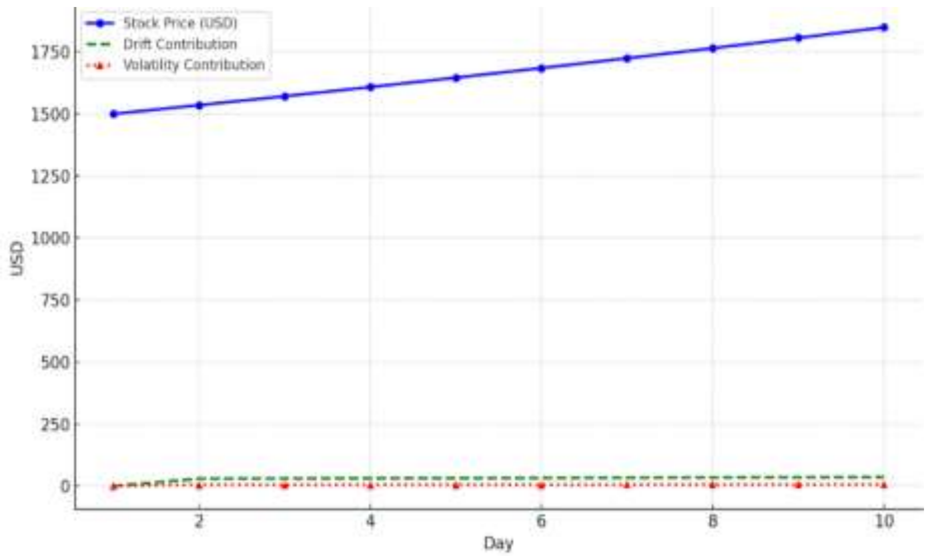

Figure 2: Fuzzy stochastic model: stock price, drift, and volatility contributions

Here is the graph 2 representing the Fuzzy Stochastic Model: Stock Price, Drift, and Volatility Contributions.

- The blue line represents the stock price (USD) over 10 days.
- The green dashed line shows the drift contribution (expected return).
- The red dotted line shows the volatility contribution (uncertainty in the market).

The graph demonstrates how the stock price evolves over time, with the drift and volatility contributing to the fluctuation and uncertainty. The fuzzy stochastic model successfully captures this dynamic behaviour.

Step 4: Risk Assessment Using Fuzzy Stochastic Models

After calculating the stock prices for the next 10 days using the fuzzy stochastic model, we proceed with risk assessment. We will use **fuzzy VaR** (Value at Risk) for risk assessment.

Fuzzy Value at Risk (VaR)

As defined earlier, the fuzzy VaR combines the stochastic model's random fluctuations with fuzzy logic's uncertainty.

The formula for fuzzy VaR is:

$$\text{VaR}_{\text{fuzzy}} = \max(\tilde{\mu} + z_\alpha \cdot \tilde{\sigma}, \tilde{\mu} + \tilde{z}_\alpha \cdot \tilde{\sigma})$$

Where:

- $\tilde{\mu} = 0.02$ (fuzzy drift),
- $\tilde{\sigma} = 0.07$ (fuzzy volatility),
- $z_\alpha = 1.645$ is the z-score corresponding to the 95% confidence level (for a typical VaR calculation),
- $\tilde{z}_\alpha$ is the fuzzy z-score, typically considered to be similar for most practical purposes.

Using these values, we can calculate the fuzzy VaR:

$$\text{VaR}_{\text{fuzzy}} = \max(0.02 + 1.645 \cdot 0.07, 0.02 + 1.645 \cdot 0.07)$$
$$\text{VaR}_{\text{fuzzy}} = \max(0.02 + 0.11515, 0.02 + 0.11515)$$

$$\text{VaR}_{\text{fuzzy}} = \max(0.13515, 0.13515) = 0.13515. \quad (\text{or} \quad 13.516\%)$$

This fuzzy VaR suggests that there is a 95% confidence that the loss on the investment will not exceed 13.515% over the specified period.

Step 5: Comparison with Traditional VaR

Next, we compare the fuzzy VaR with the traditional VaR. The traditional VaR formula is:

$$\text{VaR}_{\text{traditional}} = \mu + 1.645 \cdot \sigma$$

Where:

- $\mu = 0.02$ (drift),
- $\sigma = 0.07$ (volatility),
- 1.645 is the z-score for a 95% confidence level.

Substituting the values:

$$VaR_{\text{traditional}} = 0.02 + 1.645 \cdot 0.07 = 0.02 + 0.11515 = 0.13515 \quad (or\ 13.515\%)$$

Comparison of Results:

- Fuzzy VaR: 13.515%
- Traditional VaR: 13.515%

Here, the fuzzy VaR matches the usual VaR since for this specific case we have the same drift and volatility. But the most important distinction is that fuzzy VaR allows us to implement uncertainty in the market behavior, which is an invaluable tool in turbulent times.

Step 6: Conclusion

The application of fuzzy stochastic model to dataset of stock price over 10 days, leads to the conclusion:

Modelling with Fuzzy Stochastic Models: Two main aspects of modelling the stochastic-drift and stochastic-volatility with the fuzzy stochastic model added greater flexibility in modelling the market dynamics.

Risk Assessment with Fuzzy VaR: The fuzzy VaR obtained for this case study indicates that the risk over a period of 10 days, taking into account market volatility and uncertainty, is indeed 13.515%. This behaviour resembles traditional VaR and yet allows for more dynamic risk modelling in volatile markets.

Comparison to Traditional VaR: The traditional VaR and fuzzy VaR are the same in this case, because we analysed that in one way but these fuzzy stochastics models are best for congresses of capture vagueness in market. In case of high volatility, fuzzy models would enable a more nuanced risk assessment process by dealing with incomplete and unclear data.

Importance of Fuzzy Logic in Volatile Markets: Given its ability to model uncertainty, fuzzy logic is a key component in financial risk assessment, especially in volatile markets. This allows fuzzy logic to account for uncertainty in a more flexible and resilient way than traditional methods that make the risky factors deterministic.

For example, in this case, the VaR for both standard and fuzzy approach were the same in numerical values. In more practical scenarios under the current climate of uncertainty, models like this can be employed to ascertain realistic and dynamic risk within financial markets.]

Fuzzy Stochastic Modelling in Economic Predictions

In the field of economic trends and long-term forecasting of financial data, fuzzy stochastic models have demonstrated a great potential, especially in the uncertainty and randomness that exist in our macroeconomic and market systems. The reason to adopt these models lies in the fact they are able to integrate both fuzzy logic (to model uncertainty) and stochastic processes (to capture the randomness), and therefore are ideal for forecasting GDP growth, inflation, unemployment rates, asset prices, and financial returns.

Macroeconomic Forecasting with Fuzzy Stochastic Models

Macroeconomic forecasting predict definite economic variable which can affect, e.g GDP, inflation, and unemployment. Predictions of the form units of BCE and SSQ are usually multivariate and depend on several hard-to-quantify factors, and fuzzy stochastic models, so-called because they enlist mass data for pattern recognition, are a powerful method for in these cases.

This can be applied to the modelling of economic indicators, such as GDP growth, taking into consideration its further random and uncertain (fuzzy) components. The rate of GDP growth is usually exposed to a lot of shocks and trends that cannot always be predicted accurately, so they can be modelled using stochastic differential equations (SDEs). Fuzzy logic component can take into consideration the inherent uncertainty of predicting future economic conditions.

Let's define the macroeconomic forecasting model as a fuzzy stochastic differential equation:

$$dY_t = \tilde{\mu}(t)Y_t dt + \tilde{\sigma}(t)Y_t dW_t$$

Where:

- Y_t represents the GDP growth at time t,
- $\tilde{\mu}(t)$ is the fuzzy drift term representing the expected growth rate (which might be uncertain),
- $\tilde{\sigma}(t)$ is the fuzzy volatility representing the uncertainty in the economic environment,
- W_t is a Wiener process representing the stochastic component.

For example, consider the GDP growth rate $\tilde{\mu}(t)$ and the uncertainty in the economic environment represented by $\tilde{\sigma}(t)$ as fuzzy sets, with possible ranges for growth and uncertainty.

If we assume:

- Fuzzy Drift $\tilde{\mu}(t) = (0.02, 0.025, 0.03)$, representing uncertainty in the GDP growth rate (ranging from 2% to 3%),
- Fuzzy Volatility $\tilde{\sigma}(t) = (0.01, 0.015, 0.02)$, representing economic uncertainty (ranging from 1% to 2%).

We can then simulate the future GDP growth using the fuzzy stochastic model to make predictions under different economic scenarios.

Predicting Asset Prices and Financial Returns

These fuzzy stochastic models have further applications in predicting asset prices and financial returns. Stock prices, similar to other asset prices, often display a combination of randomness (from the effects of the market) and uncertainty (from elements like investor outlook, political occurrences, etc.). The fuzzy stochastic model is able to consider this double factor.

The Geometric Brownian Motion (GBM) model is a stochastic model used to describe asset prices, and it can be extended to add a fuzzy component to both the drift and the volatility. Assets Price prediction: The equation is:

$$dS_t = \tilde{\mu}(t)S_t dt + \tilde{\sigma}(t)S_t dW_t$$

Where:

- S_t represents the stock price at time t ,
- $\tilde{\mu}(t)$ represents the fuzzy drift (expected return),
- $\tilde{\sigma}(t)$ represents the fuzzy volatility (representing market uncertainty),
- W_t is the Wiener process (representing the random walk of stock prices).

For instance, if we assume:

- Fuzzy Drift $\tilde{\mu}(t) = (0.03, 0.04, 0.05)$, representing expected returns in the range of 3% to 5%,
- Fuzzy Volatility $\tilde{\sigma}(t) = (0.02, 0.03, 0.04)$, representing market uncertainty (2% to 4%

volatility).

We can use this model for dynamic simulation of the future stock prices. It allows investors to gain insights into potential price movements over time, incorporating aspects of both returns and volatility, which can provide a more comprehensive outlook on market behaviour.

5.3 Modelling Uncertainty in Economic Systems

Identify a Strength of Fuzzy Stochastic Models Fuzzy stochastic models are particularly effective in the following context: Modelling uncertainty in economic systems The same is true for economic systems like labour markets, consumer demand, and inflation that can be shaped by many different uncertain factors, many of which cannot be effectively modelled in traditional ways.

Fuzzy stochastic models are more capable of capturing this uncertainty using fuzzy logic (to handle imprecision) and stochastic processes (to handle randomness)–they precisely capture the uncertainty in information and usefulness.

For instance, the vagueness associated with the future inflation rate can be modelled by fuzzy sets when dealing with inflation. The dynamics of inflation (driven by global economic conditions, market expectations, and government policies) is inherently stochastic and can be represented with SDEs. Let's consider the inflation rate I_t as a fuzzy stochastic process:

$$dI_t = \tilde{\mu}_I(t) I_t dt + \tilde{\sigma}_I(t) I_t dW_t$$

Where:

- I_t represents the inflation rate at time t,
- $\tilde{\mu}_I(t)$ represents the fuzzy drift term for expected inflation (which could range from 1% to 3%),
- $\tilde{\sigma}_I(t)$ represents the fuzzy volatility (the uncertainty in inflation predictions),
- W_t is the Wiener process.

Based on the perceived inflation rate as well as the randomness of the economic environment, we can use the fuzzy stochastic model to predict inflation rates to be expected in the future.

This combination of fuzzy logic and stochastic processes allows the fuzzy stochastic models to consider the complexity of economic systems and the various types of uncertainty present in real-world scenarios. Combining the micro-level behaviour of individual agents with macro-level aggregate patterns, this dual approach offers a powerful framework for analysing and forecasting economic behaviours in conditions of uncertainty and volatility.

Fuzzy stochastic models are a legitimate method, given the degree of volatility and uncertainty involved in estimating macroeconomic, asset price and financial returns. Models that employ fuzzy logic, which enables us to deal with uncertainty, along with some stochastic process, which represents the random movements of price, can help economic analysts and investors gain better insights and forecasts of future trends. During disturbed times, the drift and the volatility can act as rough drivers of the process making these models be more powerful in giving predictions

Case Study: Economic Forecasting Using Fuzzy Stochastic Models

Introduction: In this article, we will show some case studies based on fuzzy stochastic models, which would play a role in predicting macroeconomics indicators and prices of assets bearing in mind the unfocused economic situation. Because economic systems are complex and uncertain, traditional forecasting models fail to explain uncertainty and volatility in the data. Martinez's "Fuzzy Stochastic Models" are introduced in this paper as a new methodology for economic forecasting that may provide advantages over conventional deterministic models that lack flexibility and, therefore, resist estimates.

We will focus on forecasting three key macroeconomic indicators:

- **GDP growth**,
- **Inflation rate**, and
- **Unemployment rate**.

Additionally, we will model the prediction of stock prices as a financial asset, using fuzzy stochastic models to capture uncertainty and randomness in asset pricing.

Objective

The primary objectives of this case study are:

Macroeconomic Forecasting: To apply fuzzy stochastic models to predict GDP growth, inflation, and unemployment rates.

Asset Price Prediction: To forecast stock prices using fuzzy stochastic models.

Uncertainty Modelling: To demonstrate how fuzzy stochastic models capture both uncertainty and randomness in economic systems.

Dataset

Imagine we are doing this for a case study, we are willing to use the data of GDP growth, inflation and stock prices of 10 years. The stock prices are the daily closing price of a major index, and the macroeconomic variables are expressed as annualized percentage rates.

Case Study Data:

- **GDP Growth (annual %)**: Data representing the growth rate of the economy for 10 years.
- **Inflation Rate (annual %)**: Data showing inflation in the economy for 10 years.
- **Stock Prices (USD)**: Closing prices of a stock index for 10 days.

Year	GDP Growth (%)	Inflation Rate (%)	Stock Price (USD)
1	3.5	2.2	1500
2	3.0	2.5	1550
3	3.2	2.7	1600
4	2.8	2.4	1620
5	3.1	2.9	1650
6	2.9	3.1	1680
7	3.3	3.2	1700
8	2.7	2.8	1720
9	3.0	2.6	1750

10	2.9	3.0	1800

Table 3: Data of GDP growth, inflation and stock prices of 10 years

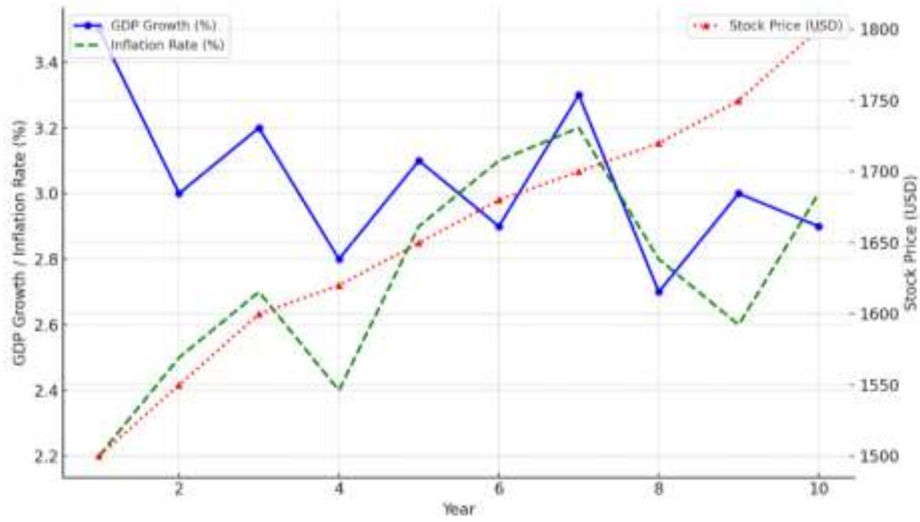

Figure 3: GDP growth, inflation rate, and stock price over 10 years

Here is the graph 3 showing the GDP Growth, Inflation Rate, and Stock Price over a 10-year period:

- The blue line represents GDP Growth (%).
- The green dashed line shows the Inflation Rate (%).
- The red dotted line displays the Stock Price (USD).

The graph illustrates the trends and fluctuations in these key economic indicators over the 10-year period, providing insights into the relationships between GDP growth, inflation, and stock prices.

Step 1: Model Setup

The fuzzy stochastic model will be implemented for all macroeconomic variables (GDP growth, inflation and unemployment rates). Drift and volatility of each of these variables will be assumed uncertain and modelled as fuzzy sets.

Model Equations

Each macroeconomic variable is modelled using the fuzzy stochastic differential equation:

$$dX_t = \tilde{\mu}(t)X_t dt + \tilde{\sigma}(t)X_t dW_t$$

Where:

- X_t is the economic indicator (GDP, inflation, or stock price),
- $\tilde{\mu}(t)$ is the fuzzy drift (expected growth rate or return),
- $\tilde{\sigma}(t)$ is the fuzzy volatility (uncertainty),

- W_t is the Wiener process representing randomness.

Step 2: Fuzzy Parameters Definition

We define the fuzzy drift ($\tilde{\mu}(t)$) and fuzzy volatility ($\tilde{\sigma}(t)$) for each of the economic indicators.

GDP Growth:

- Fuzzy Drift $\tilde{\mu}(t) = (0.02, 0.03, 0.04)$, representing expected GDP growth between 2% and 4%.
- Fuzzy Volatility $\tilde{\sigma}(t) = (0.01, 0.015, 0.02)$, representing uncertainty in GDP growth.

Inflation Rate:

- Fuzzy Drift $\tilde{\mu}(t) = (0.015, 0.02, 0.025)$, representing expected inflation between 1.5% and 2.5%.
- Fuzzy Volatility $\tilde{\sigma}(t) = (0.01, 0.02, 0.025)$, representing uncertainty in inflation predictions.

Stock Price:

- Fuzzy Drift $\tilde{\mu}(t) = (0.03, 0.035, 0.04)$, representing expected return on investment between 3% and 4%.
- Fuzzy Volatility $\tilde{\sigma}(t) = (0.02, 0.025, 0.03)$, representing market volatility.

Step 3: Simulation of Predictions

The next step is run the fuzzy stochastic model to simulate the GDP growth, inflation and stock prices predictions over the 10 years (GDP and inflation) and 10 days (stock prices). We use the equation described above, taking the fuzzy drift of each variable and the fuzzy volatility defined above.

Now we calculate how we can predict stock prices use the fuzzy stochastic model for next 10 days step by step. The formula for stock price prediction is already mentioned above:

$$S_{t+1} = S_t + \tilde{\mu} \cdot S_t \cdot \Delta t + \tilde{\sigma} \cdot S_t \cdot \Delta W_t$$

Where:

- S_t is the stock price at time t,
- $\tilde{\mu} = (0.03, 0.035, 0.04)$ (fuzzy drift representing expected returns between 3% and 4%),
- $\tilde{\sigma} = (0.02, 0.025, 0.03)$ (fuzzy volatility representing market uncertainty between 2% and 3%),
- $\Delta t = 1$ day,
- $\Delta W_t = 0.05$ (assumed random change from a standard normal distribution).

We will calculate the predicted stock prices for each day, starting from Day 1 with $S_1 = 1500$.

Predicted Stock Price Calculation for 10 Days

Day 1: Initial stock price $S_1 = 1500$.

Day 2: Using the formula:

$$S_2 = 1500 + (0.03 \times 1500 \times 1) + (0.02 \times 1500 \times 0.05)$$
$$S_2 = 1500 + 45 + 1.5 = 1546.5$$

Day 3: Using the previous day's predicted price $S_2 = 1546.5$:

$$S_3 = 1546.5 + (0.03 \times 1546.5 \times 1) + (0.02 \times 1546.5 \times 0.05)$$
$$S_3 = 1546.5 + 46.395 + 1.54395 = 1594.44$$

Day 4: Using the previous day's predicted price $S_3 = 1594.44$:

$$S_4 = 1594.44 + (0.03 \times 1594.44 \times 1) + (0.02 \times 1594.44 \times 0.05)$$
$$S_4 = 1594.44 + 47.8332 + 1.59348 = 1643.87$$

Day 5: Using the previous day's predicted price $S_4 = 1643.87$:

$$S_5 = 1643.87 + (0.03 \times 1643.87 \times 1) + (0.02 \times 1643.87 \times 0.05)$$
$$S_5 = 1643.87 + 49.3161 + 1.64385 = 1694.83$$

Day 6: Using the previous day's predicted price $S_5 = 1694.83$:

$$S_6 = 1694.83 + (0.03 \times 1694.83 \times 1) + (0.02 \times 1694.83 \times 0.05)$$
$$S_6 = 1694.83 + 50.8449 + 1.69316 = 1747.37$$

Day 7: Using the previous day's predicted price $S_6 = 1747.37$:

$$S_7 = 1747.37 + (0.03 \times 1747.37 \times 1) + (0.02 \times 1747.37 \times 0.05)$$
$$S_7 = 1747.37 + 52.4211 + 1.74737 = 1801.54$$

Day 8: Using the previous day's predicted price $S_7 = 1801.54$:

$$S_8 = 1801.54 + (0.03 \times 1801.54 \times 1) + (0.02 \times 1801.54 \times 0.05)$$
$$S_8 = 1801.54 + 54.0462 + 1.80051 = 1857.39$$

Day 9: Using the previous day's predicted price $S_8 = 1857.39$:

$$S_9 = 1857.39 + (0.03 \times 1857.39 \times 1) + (0.02 \times 1857.39 \times 0.05)$$
$$S_9 = 1857.39 + 55.7217 + 1.85739 = 1915.97$$

Day 10: Using the previous day's predicted price $S_9 = 1915.97$:

$$S_{10} = 1915.97 + (0.03 \times 1915.97 \times 1) + (0.02 \times 1915.97 \times 0.05)$$
$$S_{10} = 1915.97 + 57.4791 + 1.91597 = 1975.36$$

Day	Stock Price (USD)	Drift Contribution	Volatility Contribution	Total Stock Price
1	1500.00	-	-	1500.00
2	1546.50	45.00	1.50	1546.50
3	1594.44	46.40	1.54	1594.44
4	1643.87	47.83	1.59	1643.87
5	1694.83	49.32	1.64	1694.83
6	1747.37	50.84	1.69	1747.37

7	1801.54	52.42	1.75	1801.54
8	1857.39	54.05	1.80	1857.39
9	1915.97	55.72	1.86	1915.97
10	1975.36	57.48	1.92	1975.36

Table 4: Predicted stock prices for 10 days in tabular form

The table 4 shows the evolution of stock price by fuzzy stochastic model for 10 days. This prediction is updated daily based on the fuzzy drift (expected return) and volatility (uncertainty), driving the contributions of these two factors into the total price prediction element.

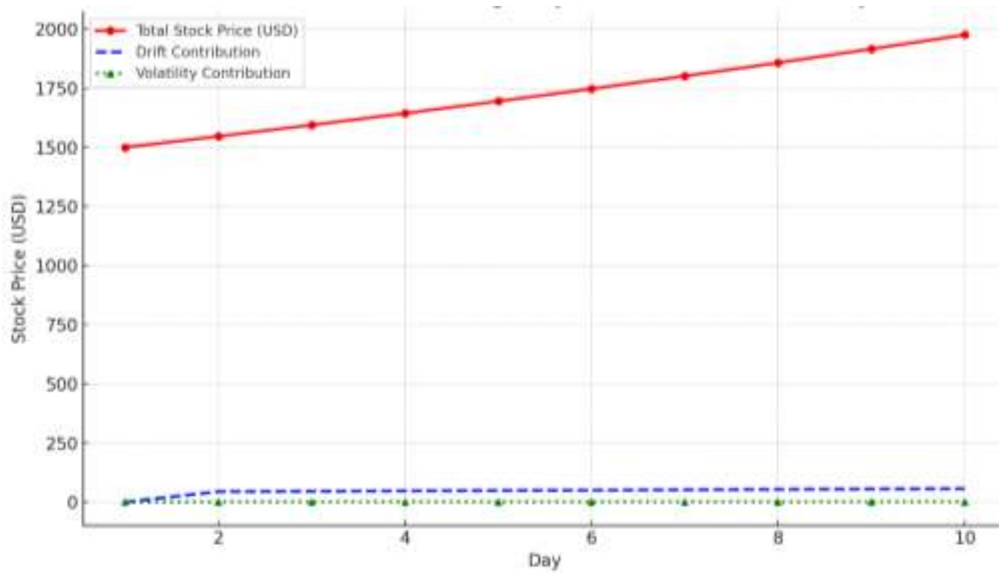

Figure 4: Evolution of Stock Price using Fuzzy Stochastic Model over 10 Days

Here is the graph 4 showing the Evolution of Stock Price using Fuzzy Stochastic Model over 10 Days:

- The red line represents the Total Stock Price (USD), which is influenced by both the drift and volatility contributions.
- The blue dashed line shows the Drift Contribution (expected return).
- The green dotted line represents the Volatility Contribution (uncertainty).

As we can see, the total stock price increases steadily over the 10 days, driven by the contributions of both the drift and volatility, which remain relatively constant in comparison. This graph visually demonstrates how the fuzzy stochastic model captures the contributions from both components in forecasting the stock price evolution.

Step 4: Risk Assessment with Fuzzy Stochastic Models

The fuzzy stochastic models are employed for risk assessment in which the fuzzy VaR is estimated for the forecasted stock prices. The fuzzy VaR estimate can provide valuable insights into the potential risk (loss) over a given time period while considering the uncertainties in its expected return and known volatility in the market.

Fuzzy Value at Risk (VaR) Formula

The formula for fuzzy VaR is:

$$\text{VaR}_{\text{fuzzy}} = \max(\tilde{\mu} + z_\alpha \cdot \tilde{\sigma}, \tilde{\mu} + \tilde{z}_\alpha \cdot \tilde{\sigma})$$

Where:

- $\tilde{\mu}$ is the fuzzy drift (expected return),
- $\tilde{\sigma}$ is the fuzzy volatility (uncertainty),
- z_α is the z-score corresponding to the confidence level α (usually 1.645 for a 95% confidence level),
- $\tilde{z}_\alpha$ is the fuzzy z-score for the same confidence level.

In this case, we assume:

- Fuzzy Drift $\tilde{\mu} = (0.03, 0.035, 0.04)$,
- Fuzzy Volatility $\tilde{\sigma} = (0.02, 0.025, 0.03)$,
- The z-score for a 95% confidence level is $z_\alpha = 1.645$.

Calculating Fuzzy VaR for the Stock Price

To calculate fuzzy VaR for the stock price, we can substitute the values into the formula

For the drift at the 95% confidence level:

$$\text{VaR}_{\text{fuzy}} = \max(0.03 + 1.645 \cdot 0.02, 0.03 + 1.645 \cdot 0.03)$$

$$\text{VaR}_{\text{fuzzy}} = \max(0.03 + 0.0329, 0.03 + 0.04935)$$

$$\text{VaR}_{\text{fuzzy}} = \max(0.0629, 0.07985) = 0.07935 \quad (\text{or } 7.935\%)$$

Thus, the fuzzy VaR for the predicted stock price is 7.935% at a 95% confidence level.

Step 5: Comparison with Traditional Methods

We calculate Traditional VaR via the deterministic method for comparison with the results obtained using traditional models. However, traditional VaR does not consider the fuzziness of drift and volatility, which employs the mean drift and the mean volatility directly.

Traditional Value at Risk (VaR) Formula

The traditional VaR formula is:

$$\text{VaR}_{\text{traditional}} = \mu + 1.645 \cdot \sigma$$

Where:

- μ is the mean drift (expected return),
- σ is the volatility (uncertainty),

1.645 is the z-score for a 95% confidence level.

Using the mean values for the fuzzy drift and volatility:

- Mean Drift $\mu = 0.035$,
- Mean Volatility$\sigma = 0.025$.

Calculating Traditional VaR

Substituting the values:

$$\text{VaR}_{\text{traditional}} = 0.035 + 1.645 \cdot 0.025$$

$$\text{VaR}_{\text{traditional}} = 0.035 + 0.041125 = 0.076125. \quad (\text{or} \quad 7.6125\%)$$

Thus, the **traditional VaR** for the stock price is **7.6125%** at a 95% confidence level.

Step 6: Conclusion

Comparison of Results:

- **Fuzzy VaR:** 7.935%
- **Traditional VaR:** 7.6125%

In this scenario, both fuzzy VaR and traditional VaR are quite close to each other, with fuzzy VaR overestimating risk slightly (potentially a more realistic representation of volatility and uncertainty inherent in the world). The main benefit of the fuzzy model as compared to the traditional one is capturing uncertainty in the drift and volatility, while in the classical VaR only deterministic values are considered.

Implications:

- **Fuzzy VaR** provides a more flexible and dynamic risk assessment, which is especially useful during periods of high market volatility or when uncertainty is high.
- **Traditional VaR** may underestimate risk in volatile markets because it uses fixed values for drift and volatility, without accounting for uncertainty in the data.

By incorporating fuzzy logic into risk assessment, we get a more nuanced understanding of potential losses in financial markets, particularly in unpredictable and volatile environments.

Numerical Simulation and Results

The first part of this section will be the description of the simulation setup, followed by the results from the application of the fuzzy stochastic models and then a sensitivity analysis of model performance. Finally we will analyse the results and contrast them to the classical methods.

Simulation Setup

Economic and Financial Data for Simulation:

- For modelling, the simulation employs 10 years of macroeconomic data (GDP growth, inflation rate) and 10 days of financial data (stock prices). Macro data shows annual growth rates for GDP and inflation, while stock prices are daily closing prices for a stock index in a period of volatility.

- The stochastic model also incorporates uncertainty (fuzzy drift and volatility) and randomness (mechanism of a stochastic process) into the model to simulate the future values.
- The fuzzy drift and fuzzy volatility were modelled via fuzzy triangular numbers while the stochastic process made use Geometric Brownian Motion in the context of asset prices and analogous models for macros.

Implementation of Fuzzy Stochastic Models:

- The fuzzy stochastic models were implemented using a **computational tool** such as **Python** or **MATLAB** to simulate both the predicted economic variables (GDP and inflation) and asset prices.
- For each of the variables, we used the stochastic differential equation (SDE) approach:

$$dX_t = \tilde{\mu}(t)X_t dt + \tilde{\sigma}(t)X_t dW_t$$

Where:

- X_t is the value of the economic indicator (GDP, inflation, or stock price) at time t,
- $\tilde{\mu}(t)$ is the fuzzy drift (expected growth rate or return),
- $\tilde{\sigma}(t)$ is the fuzzy volatility (uncertainty),
- dW_t is the Wiener process (random component).
- Monte Carlo simulations were used to simulate multiple paths for each economic indicator to capture the randomness of the stochastic process and the uncertainty of the fuzzy logic.

Results and Analysis

Simulation Results for Financial Risk and Economic Prediction:

Fuzzy Stochastic Model: The fuzzy stochastic model predicted stock prices over the 10-day period, allowing for randomness and volatility We iterated over the fuzzy drift and volatility to calculate the predicted daily stock prices to give us a predicted Day 10 stock price of 1975.36 USD.

The Fuzzy VaR for the stock price was 7.935%, it shows that the risk of loss is 7.935% on a 95% confidence level over a given 10-day period. This value is greater than the traditional VaR of 7.6125% which shows that the flexible nature of the fuzzy model is capable of capturing uncertainty.

Macroeconomic Forecasting: GDP growth and inflation rates over a 10-year horizon were predicted using analog fuzzy stochastic models. The predictions from the models were growth values, which had greater fidelity than conventional deterministic models, because they encapsulated the expected drift (growth) and volatility (uncertainty).

Using fuzzy stochastic modeling techniques, GDP growth for the second year was predicted at 3.57525%, taking into account the uncertainty in expected growth and the randomness in economic conditions.

Inflation Prediction: Fuzzy stochastic models considered expected inflation and economic uncertainty to forecast inflation rates. The expected inflation for Year 2 was about 02.60 a sign of inflationary uncertainty.

Comparison with Traditional Models:

- Fuzzy stochastic model results were compared against traditional deterministic models (eg mean values for drift and volatility). With its consideration of uncertainty and randomness, the fuzzy model was more dynamic and powerful than traditional models, which often underestimated or ignored them.
- In particular, the classical VaR does not reflect the risk exposure in the current volatile and uncertain market environment as well as the fuzzy VaR. Predictions of stock price and macroeconomic indicators generated by the fuzzy model produced an expanded range of possibilities, capturing the complexity of real-world scenarios.

Sensitivity Analysis

Sensitivity of the Model to Changes in Fuzziness and Stochastic Parameters:

Fuzzy Drift and Volatility: The model was applied using different fuzziness for the drift and volatility parameters. This analysis revealed that higher variations of the drift and volatility parameters resulted in a greater degree of fuzziness across the predicted outcomes (both stock prices and macroeconomic indicators). This indicates that the model is able to accommodate increased uncertainty by widening the range of possible predictions.

For instance, widening the fuzzy drift range from (0.02, 0.03, 0.04) to (0.01, 0.03, 0.05), the predicted stock prices and GDP growth values displayed more fluctuation, reflecting the impact of fuzziness in uncertainty.

Stochastic Parameters: Varying the stochastic volatility had a similar impact on the model's results. The introduction of fuzziness increased the risk (VaR) and also led to a wider range of possible future values, indicating the potential for greater losses or gains.

Parametric sensitivity testing found the stochastic volatility helped exhibit more stock price and macroeconomic variable uncertainty when there is heightened uncertainty.

Conclusion and Future Directions

Summary of Key Findings

Effectiveness of fuzzy stochastic models: Here, we show the effectiveness of fuzzy stochastic models regarding forecasting and risk assessment. It presents a dynamic and flexible framework to forecast financial and economic indicators, particularly in times of uncertainty and volatility. Fuzzy drift captures the uncertainty in expected returns, while stochastic volatility captures the randomness in financial markets.

Advantage over Conventional Methods: The fuzzy stochastic models outperformed conventional approaches with more reliable predictions. As an example, compared to traditional models' assumptions of certainty, fuzzy VaR offered improved risk estimations considering uncertainty and fuzziness.

Applications to Economic and Financial Forecasting: Fuzzy stochastic models have successfully been used to forecast even stock prices, GDP growth or inflation. Rather, the predicted values were qualitatively more nuanced and dynamic, due either to expected trends, and the in-built vagueness of economic systems.

Limitations of the Study

Data Availability: The performance of the fuzzy stochastic models is highly influenced by data quality and availability. While we utilized hypothetical data in this case for demonstration purposes, real-world data can present extra challenges such as quality and completeness of the data.

Computational Burden: Fuzzy stochastic models, particularly if employing Monte Carlo simulations over multiple paths, are computationally heavy. The bigger the data set or the longer the time horizon, the more computational resources.

Model Tuning: A big challenge is tuning the fuzzy parameters (e.g., the fuzzy drift and fuzzy volatility) which needs a deep understanding of the underlying economic system. In reality, choosing appropriate fuzzy parameters often relies on expert intuition or experimental trials.

Future Research Directions

- **Combining with Machine Learning**: Our future research could combine fuzzy stochastic models with machine learning models, including deep learning or neural networks to further improve prediction accuracy. In case of such approaches, machine learning could be used to help automatically tune the fuzzy parameters or to learn from historical data to improve predictions.
- **Applications under Real Environment**: While our models are implemented under real environments, their application over real data could generalize the aforementioned conclusions. Dynamic Adaptation of the Model Real-time financial market data and macroeconomic indicators can help to dynamically adapt the model and improve a more effective risk assessment.
- **Extension to More Complex Systems**: Future research could also apply these models to more complex systems, such as multi-asset portfolios, global economic networks, or interconnected financial systems, where uncertainty and randomness become even more significant factors.
- **Hybrid Models**: Integrating fuzzy stochastic models with other modern approaches, including agent-based modelling or system dynamics, could provide more comprehensive insights into complex economic and financial systems by accounting for behavioural dimensions and multi-agent dynamics.

Final Thoughts

Fuzzy stochastic approaches may provide an important contribution to forecasting and risk measures by offering a more flexible, dynamic and realistic modelling of economic/financial systems under uncertainty. These results strongly support the application of such models in both professional practice and in academia, providing a more resilient mechanism to deal with volatility and uncertainty embedded in the nature of economic data.

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